Principles of electrography applied to self-potential electrokinetic sources and hydrogeological applications

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[1] The electrical potential field passively recorded at the ground surface of the Earth (and termed self-potential) can be analyzed to determine the shape and the depth of the piezometric surface. The coupling between hydraulic flow and electrical current density is electrokinetic in nature. The electrokinetic coupling coefficient entering into the integral equation relating the depth of the water table to self-potential signals is analyzed for various types of porous materials. It is simply related to the electrical conductivity of the pore water. In steady state conditions each element of the water table can be seen as an elementary dipole with an inclination locally perpendicular to the water table and strength proportional to the water table elevation. Then, we propose three methods to obtain the shape and range of possible depths of the water table from the study of the self-potential distribution recorded at the ground surface. The nonuniqueness of the solution is removed if one knows either the electrokinetic coupling coefficient or the water table at one location and under the assumption of the spatial homogeneity of the electrokinetic coupling coefficient. Two field cases are discussed to show the success of the proposed methods for estimating the shape and depth of the water table at two different scales of investigations. They concern the study of self-potential signals associated with the shape of the water table in the vicinity of a pumping well and in the flank of the Kilauea volcano.

INDEX TERMS: 1832 Hydrology: Groundwater transport; 5109 Physical Properties of Rocks: Magnetic and electrical properties; 5139 Physical Properties of Rocks: Transport properties; 5114 Physical Properties of Rocks: Permeability and porosity; KEYWORDS: self-potential, hydraulic charge, water table, electrokinetic, streaming potential, tomography


1. Introduction

[2] Self-potential surveys are conducted by mapping with nonpolarizable electrodes the quasi-static natural electrical field at the ground surface of the Earth. These electrical signals represent the ground surface electrical field signature of some charging mechanism occurring at depth [e.g., Fournier, 1989; Patella, 1997a, 1997b]. It is widely accepted that, in absence of strong oxidoreduction processes and after correction of telluric currents, the main contributor to self-potential anomalies corresponds to an electrokinetic conversion of the groundwater flow [Aubert and Atangana, 1996; Birch, 1998; Patella, 1997a; Revil et al., 1999a, 1999b]. Indeed, when water flows inside the connected pore space of a porous medium, it drags along part of the excess of the electrical charge located in the close vicinity of the mineral/water interface, in what is called the electrical triple layer. A description of this phenomenon will be provided later in this paper (in section 3.1).

[3] In the last 15 years, there has been a considerable reawakening interest in the application of electrokinetic phenomena in geohydrology. Applications concern the study of water leakage from dams [Al-Saigh et al., 1994; Trique et al., 1999], groundwater flow in geothermal fields and active volcanoes [Ishido, 1989; Tanaka, 1993; Hashimoto and Tanaka, 1995; Patella, 1997b], and groundwater flow associated with topographic variations of the water table [Fournier, 1989; Birch, 1993, 1998].

[4] Because electrokinetic conversion produces a detectable electrical field at the ground surface, it should be possible to invert the pattern of this electrical field to get out some information about the pattern of fluid flow. We call “electrography” such an inversion process applied to self-potential signals in the quasi-static limit. This procedure opens exciting perspective in geohydrology like, for example, to determine hydraulic conductivity around a pumping well. Indeed, if one knows the pressure distribution everywhere during a pumping test, well established inverse groundwater modeling allows to determine the distribution of the hydraulic conductivity and storativity around the borehole [e.g., Cooley, 1977, 1979; Neuman and Yakowitz, 1979; Neuman et al., 1980; Yeh and Yoon, 1981; Kitanidis and Vomvoris, 1983; Yeh et al., 1983; Loaiciga and Marino, 1987]. However, the piezometric surface is usually not well constrained due to a lack of observation wells. In addition, the presence of observation wells disturbs the distribution of the hydraulic head. Therefore a nondestructive and cheap

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method to determine the shape of the piezometric surface would be very welcome in geohydrology. The purpose of this paper is to present self-potential data inversion for estimating the depth of the water table and to show some applications of electrography.

We cover below the various aspects of this method including (1) the analysis of the physical foundations of the electrokinetic conversion in the field (section 2 and Appendix A), (2) the microscopic roots of this phenomenon and its sensitivity to groundwater flow in the field (section 3), (3) the development of various methods to determine the shape of the water table from self-potential data (section 4), and (4) the application of these methods to two case studies covering two very different depths of investigation of the water table (section 5).

2. Representation of the Self-Potential Field

We consider the ground to be formed by a piecewise uniform conducting space comprising regions \( \Omega_i \) with constant electrical conductivities \( \sigma_i \) where \( i = 0, 1, 2, 3, \ldots \). The region \( \Omega_0 \), confined by the surface \( \partial \Omega \), represents the porous volume in which fluid flow takes place (Figure 1). As shown in Appendix A, fluid flow is responsible for a dipolar charge separation through electrokinetic coupling. This charge separation is generally (but not necessarily) positive in the flow direction [e.g., Pengra et al., 1999; Revil et al., 1999b]. At the micro-scale of the porous rock, the electrical field is due to the drag of the excess of charge contained in the vicinity of the pore-water-mineral interface by the pore water flow see Figure 2 and section 3). This local charge separation generates a volume distribution of current dipole moment inside the source region \( \Omega_0 \). Each dipole points locally in the direction of groundwater flow. In Appendix A, we show that, in steady state conditions, this boundary-value problem can be recast in an equivalent potential problem in which the piezometric surface \( \partial \Omega \) (the water table) carries a dipole layer of charge with a strength proportional to the piezometric head. As developed in Appendix A, the electrical potential \( \varphi \) (in Volts) measured at the observation station \( P \) located outside the source volume \( \Omega_0 \) (usually \( P \) is located at the ground surface) is:

\[
\varphi(P) = \frac{C}{4\pi} \int_{\partial \Omega} (h - h_0) \left( \frac{r \cdot n}{r^3} \right) dS + \frac{1}{4\pi} \int_{\Omega} \frac{E \cdot \nabla \varphi}{\rho} dV, \tag{1}
\]

where \( n \) is the outward normal to the water table at the source point \( M \) shown in Figure 1, \( dV \) is a volume element of the ground, \( dS \) is a surface element of the water table around \( M \), \( h \) and \( h_0 \) are the hydraulic heads at source point \( M \) in a reference level, respectively (Figure 1), and \( r \equiv MP \) (\( r \) is the vector from source point \( M \) to observation station \( P \)), and \( E = -\nabla \varphi \) is the electrical field produced in the ground through the electrokinetic coupling. The material properties entering into (1) are the electrical resistivity (in \( \Omega \cdot m \)) \( \rho = 1/\sigma \) where \( \sigma \) is the electrical conductivity (in \( S \cdot m^{-1} \)), and the electrokinetic coupling coefficient \( C' = \partial \varphi/\partial h \) (in \( V \cdot m^{-1} \)) described further in section 3. The first term of (1) represents the primary source term associated with the groundwater flow whereas the second term corresponds to secondary sources associated with electrical resistivity contrasts in the ground. The primary source term is such that each element of the water table acts as a small dipole of strength \( C' (h - h_0) \). All these dipoles contribute to the self-potential signals recorded at the observation station \( P \) with strength depending also on the distance between each dipole and the observation station \( P \).

[7] The purpose of electrography is to recover the depth and shape of the water table from (1) using self-potential measurements performed at the ground surface and electrical resistivity tomography to remove the secondary source terms. However, such a way to proceed is not trivial and we prefer to show in this paper that despite the fact these secondary sources exist, a first-order assumption to neglect them provides already quite good results. Consequently, we will ignore the effects of spatially varying electrical conductivity due to geological heterogeneity and water content variations and we focus below on the first term associated with the electrokinetic coupling. This is a good approximation when the unsaturated zone appears quite uniformly resistive. The inversion procedures developed in section 4 consist in finding a set of possible
depths and shapes for the water table, and then use either the hydraulic head at least at one location or the value of the coupling term $C'$ (investigated in section 3) to constrain the solution of the problem.

[8] In addition to the inversion approaches proposed in section 4, we have developed a semiempirical approach of the problem linking the depth of the water table to the measured self-potential signals. For the sake of simplicity, we consider the case of the self-potential signals associated with a pumping test in steady state conditions like the one shown in Figure 1b. Pumping in a well causes a depression cone of the (unconfined) water table plus a sink term in the groundwater continuity equation. The shape of the water table reflects the variations of the pore fluid pressure in the vicinity of the pumping well in response to pumping. Let us consider the difference of electrical potential between a point located far from the pumping well, in an undisturbed area (no water flow), and a point located in the depression cone below the observation station P. Far from the pumping well, we assume that the reference potential is taken as zero. According to the definition of the coupling coefficient $C'$, the electrical potential at an observation point P is given approximately by $\varphi(P) = (h - h_0)C'$. For the case shown in Figure 1b, this yields $\varphi(P) = -C'(e - h) + C'(e - h_0)$, where $(e - h)$ represents the depth of the water table below the observation point P. We prefer to rewrite this semiempirical law as $\varphi(P) = -K(e - h) + \varphi_0$, where $K$ (in V m$^{-1}$) is treated as an empirical parameter and $\varphi_0$ is a reference state. Various test show that we have approximately $K \approx C'/4$ for $C'$ in the coupling coefficient arising in (1).

[9] According to the simple scheme presented above, the self-potential signal would be linearly dependent on the depth of the water table. This approach and two tomographic algorithms are developed further in section 4 and applied to field data in section 5. Prior to this analysis, we focus in section 3 to the description of the electrokinetic coupling coefficient $C'$, which is used to evaluate the strength of the self-potential signals associated with a piezometric head change.

3. Sensitivity of Electrography

[10] The second part of our analysis is to provide a way to compute the order of magnitude of the electrokinetic coupling term entering into equation (1), $C'$. The knowledge of this coupling term is an important step to determine the sensitivity of the self-potential method. This coupling term can be either measured in the laboratory or calculated by measuring the contributing components in the laboratory. We show below that this parameter can be determined from an easily obtainable parameter in the field, namely the electrical conductivity of the groundwater.

3.1. Coupling Coefficient of Granular Porous Materials

[11] According to previous authors [e.g., Fournier, 1989; Aubert and Atangana, 1996; Birch, 1993, 1998], the strength of the electrokinetic conversion is poorly understood. For example, Aubert and Atangana [1996] state “the self-potential generating process is at present widely unknown, even from a qualitative point of view.” However, through the analysis made recently by Lorne et al. [1999] and Revil and Leroy [2001] and the use of equation (A8) (Appendix A), the new electrokinetic term $C'$ can be explicitly related to key-properties of the porous medium. These parameters include two textural properties (the porosity $\phi$ and a grain shape parameter $m$, generally called the Archie’s exponent) and two chemical properties of the electrical triple layer coating the minerals/water interface, the specific surface conductivity $\Sigma_{sp}$ and the $\zeta$ potential, which have been modeled into a unified framework by Revil and Leroy [2001].

[12] When a mineral is in contact with water, its reactive surface becomes charged mainly through proton exchange and sorption of cations and anions onto its surface (Figure 2). This phenomenon, which is common to all minerals including oxides, aluminosilicates, and carbonates, is responsible for the appearance of a fixed charge of density $Q_0$. The net charge density in this layer is noted $Q_0$. The net charge densities $Q_0$ and $Q_{\beta}$ are responsible for an electrical field at the microscopic scale and we note $\varphi_0$ and $\varphi_{\beta}$ the (microscopic) electrical potential on the surface and the Stern plane, respectively (Figure 2) with a reference taken in the bulk pore water just outside the electrical triple layer. The electrical field such generated is shielded at very short distance (few nanometers) from the pore water-mineral interface by the ions coming from the electrolyte. Indeed, in the Coulombic field created by the charge separation previously discussed, anions and cations of the bulk pore water are attracted or repelled depending on the sign of the charge they carry. It results in the formation of a diffuse layer of counterions (Figure 2) in which the ionic concentrations obey to Boltzman statistics. The net charge density of the electrical diffuse layer $Q_\delta$ is such that at equilibrium $Q_0 + Q_{\beta} + Q_\delta = 0$ (global electroneutrality) and the electrical potential decreases exponentially from the pore fluid.
water-mineral interface with a characteristic length scale called the Debye length. [13] When a fluid flows inside the connected pore volume, it drags part of the net charge $Q_S$ of the electrical diffuse layer. This creates, at the macroscopic scale of the representative elementary volume, a net current density $j_S$ (see Appendix A), which is the source for the electric field observed in the whole space. The $\zeta$ potential entering into equation (A3) (Appendix A) is the electrical potential of the electrical triple layer at the hydrodynamic surface where the velocity of the pore fluid equals zero (skin effect). Revil and Leroy [2001] have shown that the $\zeta$ potential can be considered as the electrical potential at the interface between the diffuse layer and the Stern layer. In addition, it was shown that the Stern layer is the locus of a Joule dissipation phenomenon called surface conductivity. Combining the model of Revil and Leroy [2001] with the definition of the sensitivity coefficient $C'$ given in Appendix A yields:

$$C' \equiv \left( \frac{\partial \varphi}{\partial h} \right)_{j=0} = \frac{\varepsilon_f \rho_f G_S}{\eta_f \sigma_f H(\zeta)}.$$ (2)

where $\varepsilon_f$ and $\rho_f$ represent the dielectric constant and bulk density of the pore water, respectively, $\sigma_f$ and $\eta_f$ represent the electrical conductivity and dynamic viscosity of the pore water, respectively, $g = 9.81 \text{ m s}^{-2}$ is the gravity acceleration, $\zeta$ is the $\zeta$ potential described above, and $H(\zeta)$ arises directly from the description of the electrical conductivity $\sigma$ of porous media, which is given by:

$$\sigma = \frac{\sigma_f}{F} H(\zeta),$$ (3)

where $F = \phi^{-m}$ is the electrical formation factor (fraction), $\phi$ is the connected porosity (fraction), $m$ is called the electrical cementation exponent (dimensionless), $\xi$ is the ratio between surface ($\sigma_S$) to pore fluid ($\sigma_f$) electrical conductivity $\xi \equiv \sigma_S/\sigma_f$ (this is sometimes called the Dukhin’s number [e.g., Szymczyk et al., 1999]), $\sigma_s = 4\Sigma_d/d$ is the surface conductivity (in S m$^{-1}$), $d$ is the mean grain diameter and $\Sigma_d$ is the specific surface conductivity expressed in Siemens [Revil et al., 1998; Revil and Leroy, 2001]. In granular materials, the application of the differential effective medium theory yields expressions for the function $H(\zeta)$, which depends only on the Dukhin’s number $\xi$ and on the formation factor $F$ [Revil et al., 2002b, and reference therein]. According to Revil et al. [2002b], $H(\zeta)$ obeys the following bounds $F \sigma_s/\sigma_f \leq H(\zeta) \leq 1$ (for clay-free rocks or soils, we recover the classical law $\sigma = \sigma_f/F$). The use of these bounds yields the following new bounds for the coupling term $C'$:

$$\frac{\varepsilon_f \rho_f G_S}{\eta_f F \sigma_s} \leq C' \leq \frac{\varepsilon_f \rho_f G_S}{\eta_f \sigma_f}.$$ (4)

The right-hand side of the inequality holds at high pore fluid ionic strength (saline groundwaters) or in clay-free porous rocks. This bound is characterized by $\sigma_f \gg \sigma_S$. It corresponds to the well-known Helmholtz-Smoluchowski formula [e.g., Lorne et al., 1999; Pengra et al., 1999]. In this limit, the coupling coefficient is independent of the texture of the porous material as long as the permeability is of course large enough to allow pore water to flow through the porous material. Mention should be made that this limit can be used in many aquifers and in the case studies to follow (section 5). In this case, the coupling coefficient is independent of the scale of measurement. [14] The left-hand side of the inequality holds for fresh groundwater in porous materials for which surface conductivity is strong (e.g., clayey materials) or when $\sigma_f \ll \sigma_S$. We observe that according to the fact that generally $\zeta < 0$ [see Lorne et al., 1999; Pengra et al., 1999; Revil and Leroy, 2001], the coupling coefficient $C'$ is generally negative in agreement with field observations as shown in section 4 (note a confusion of sign in the way Fournier [1989] defined his coupling coefficient). However, in some particular conditions of pore water chemistry and for some minerals, the sign of the $\zeta$ potential can be positive [e.g., Ishido and Mizutani, 1981] and thus the coupling coefficient $C'$ can also be positive. [15] Calculating the electrokinetic coupling term $C'$ requires the determination of the $\zeta$ potential, surface conductivity, and porosity if surface conductivity cannot be neglected. An evaluation of the intensity of $C'$ can be done using the following set of parameters, which are typical of some common fresh groundwaters in low-clay environment and using the left-hand side of (4) $\varepsilon_f \approx 80$ $\varepsilon_0$ ($\varepsilon_0 = 8.84 \times 10^{-12}$ F m$^{-1}$), $\rho_f = 10^4$ kg m$^{-3}$, $\sigma_f = 2 \times 10^{-2}$ S m$^{-1}$, $\eta_f = 8.8 \times 10^{-4}$ kg m$^{-1}$ s$^{-1}$, and $\zeta = -0.030$ V ($-30$ mV) [Revil and Leroy, 2001]. This yields $C' \approx -10$ mV m$^{-1}$. This would yield strong self-potential signals. When surface conduction dominates (e.g., in clay-rich environments), the following parameters associated with shaley sands with smectite ($\zeta = -0.03$ V ($-30$ mV), $F = 20$, and $\sigma_S = 0.1$ S m$^{-1}$) leads to $C' \approx -0.1$ mV m$^{-1}$ and therefore weak self-potential signals. Therefore the presence of a high proportion of clay with high cation exchange capacity (and therefore high surface conductivity) decreases sharply the value of the electrokinetic coupling coefficient. A clayey aquitard cannot develop strong self-potential signals. This explains the sharp reductions of the electrokinetic signals in presence of clays noticed in the literature on the basis of laboratory data and field observations [e.g., Bogoslovski and Ogilvy, 1973]. [16] Measured values of the electrokinetic coupling coefficient of various consolidated and unconsolidated porous materials are reported in Figure 3. Note the inverse relationship between the coupling coefficient and the electrical conductivity of the groundwater over more than three orders of magnitudes. A log-log correlation yields $\log_{10} C = 1.088 - 1.091 \log_{10} \sigma_f$ or $\log_{10} C = -0.921 - 1.091 \log_{10} \sigma_f$ ($r = 0.987$) where $\sigma_f$ is expressed in S m$^{-1}$. For typical groundwater conductivity, $C$ lies between $-1$ mV m$^{-1}$ and $-15$ mV m$^{-1}$. This range of values is in good agreement with the field observations discussed in section 5. Therefore, using this trend the coupling term can be determined only from the electrical conductivity of the groundwater, which is easy to measure in the field.

3.2. Sensitivity of the Method Under Field Conditions [17] Self-potential measurements are actually simple, rapid, and cheap to perform under field conditions. They require only a high-impedance millivoltmeter, insulated
sandstones (from Pengra et al. [1999]), (2) crushed Fontainebleau sandstones (from Lorne et al. [1999]), (3) clayey sandstones (from Pengra et al. [1999]), (4) carbonates (from Pengra et al. [1999]), (5) glass beads (from Pengra et al. [1999]), and (6) consolidated zeolitized volcaniclastic rock samples [Revil et al., 2002b]. A total of 83 measurements are reported. The relationship between $C$ and $C$ is given by equation (A8). Log-log correlations are given in the main text.

Figure 3. Electrokineic coupling coefficient versus pore water electrical conductivity for various types of rocks in the pH range 5.6–7 (except for the carbonates). The data correspond to (1) crushed oceanic basalts (A. Revil and D. Hermitte, unpublished work, 2001), (2) crushed Fontainebleau sandstones (from Lorne et al. [1999]), (3) clayey sandstones (from Pengra et al. [1999]), (4) carbonates (from Pengra et al. [1999]), (5) glass beads (from Pengra et al. [1999]), and (6) consolidated zeolitized volcaniclastic rock samples [Revil et al., 2002b].

single-conductor wire (preferentially protected against electromagnetic induction effects), and a set of nonpolarizable electrodes like Pb/PbCl₂ electrodes. Fabrication, design, and electrochemistry of these electrodes are described by Petiau [2000], for example. Methodologies related to field measurements and reduction of noise associated with man-made sources (e.g., power lines generate self-potential peaks, which are sometimes >100 mV). Corrections can be accomplished, for example, by filtering the self-potential measurements with specific filters based on the magnetic variations recorded with a magnetometer and a reference station for the self-potential survey. Kawakami and Takasugi [1994] have shown that such a procedure reduced the noise due to telluric currents from 30 mV to 3 mV.

For short-time monitoring field surveys (few days) and small scale operations (<1 km), the sensitivity of self-potential measurements can be very good (few tenth of millivolts). For example, Revil et al. [2002a] monitored the fluctuations of the piezometric head around a ditch during a water infiltration experiment. They showed that the sensitivity of the electrodes to self-potential variations was ~0.2 mV. Therefore self-potential signals recorded over time show extremely high sensitivity to hydraulic head variations.

In summary, the flow of groundwater in the subsoil generates a substantial electrical field, which can be recorded at the ground surface. Nowadays measurements of these signals can be performed with cheap and very stable nonpolarizing electrodes and a network of electrodes with a computer-controlled monitoring acquisition system can be used for such a purpose. This also means that, with an important number of sensors, there is enough information to invert the electrical field in order to determine the distribution of the fluid pressure in the subsurface as shown in section 4.

4. Water Table Depth and Shape

We discuss in this section three methods to determine the depth and shape of the water table from self-potential signals. There are the semiempirical relationship already discussed briefly in section 2, a tomographic algorithm based on an extension of the work of Birch [1998], and an inversion scheme of the self-potential data using the Simplex algorithm.

4.1. Semiempirical Approach

As shown in section 2, the application of the semiempirical approach yields,

$$\varphi(P) = (h - h_0)K,$$

(5)
or \( \varphi(P) = -K(e - h) + \varphi_0 \) in the case study shown in Figure 1b where \((e-h)\) represents the depth of the water table below the observation station \(P\). In this situation, the self-potential signal depends linearly on the depth of the water table. A similar empirical equation was derived by Zablocki [1978] and Jackson and Kauahikaua [1987] and used in several studies to determine the depth of the water table [see Aubert et al., 1993; Aubert and Atangana, 1996; Boubekroui et al., 1998]. In our notations, the equation used by Jackson and Kauahikaua [1987] and Aubert and Atangana [1996] is written as
\[
\varphi(P) = \frac{C}{4\pi} \int_0^L \left( h(\xi) - h_0 \right) \left( \frac{r_n}{r^2} \right) d\xi
\]
\[
\varphi(P) = \frac{C}{4\pi} \int_0^L \left[ (x_0 - z) \sin \theta + (z - h) \cos \theta \right] \left( \frac{1}{(x_0 - z)^2 + (z - h)^2} \right) d\xi
\]
where \(z(x)\) represents the ground surface topography from a reference level, \(h_0\) is the hydraulic head at the outcropping of the water table (base level, see Figure 1), \(\xi\) represents the curvilinear coordinate along the piezometric line (for 2-D cases), and \(\theta\) is the slope of the water table at the source point \(M(\xi, h)\) (Figure 1). Discretization of (7) yields:
\[
\varphi(P) = \sum_{q=1}^{Q} I_1(x - x_q, e - h_q) \Gamma_{1,q} + I_2(x - x_q, e - h_q) \Gamma_{2,q}
\]
\[
I_1(x - x_q, e - h_q) = \frac{(e - h_q)}{(x - x_q)^2 + (e - h_q)^2}
\]
\[
I_2(x - x_q, e - h_q) = \frac{(e - h_q)}{(x - x_q)^2 + (e - h_q)^2}
\]
\[
\Gamma_{1,q} = \frac{[C \sin \theta_q(h_q - h_0)] \Delta \xi}{4\pi}
\]
\[
\Gamma_{2,q} = \frac{[C \cos \theta_q(h_q - h_0)] \Delta \xi}{4\pi}
\]
where \(I_1\) and \(I_2\) represent the two scanning functions (in the terminology used by Patella [1997a, 1997b]) corresponding to the horizontal and vertical components of a dipole located at the source point \(M(x_q, h_q)\), respectively, and \(\Gamma_{1,q}\) and \(\Gamma_{2,q}\) correspond to the two components \((x-\text{and } z\text{-component})\) of the dipolar source \(q\). We define two normalized cross-correlation integrals,
\[
\eta_1(x_0, z_0) = C_1 \int_{-\infty}^{+\infty} \varphi(x) h_1(x - x_q, e - h_q) dx
\]
\[
\eta_2(x_0, z_0) = C_2 \int_{-\infty}^{+\infty} \varphi(x) h_2(x - x_q, e - h_q) dx
\]
\[
C_1 = \left[ \int_{-\infty}^{+\infty} \varphi^2(x) dx \int_{-\infty}^{+\infty} \eta_1^2(x - x_q, e - h_q) dx \right]^{-1/2}
\]
\[
C_2 = \left[ \int_{-\infty}^{+\infty} \varphi^2(x) dx \int_{-\infty}^{+\infty} \eta_2^2(x - x_q, e - h_q) dx \right]^{-1/2}
\]
where \(C_1\) and \(C_2\) are the two normalizing factors. The distributions \(\eta_1(x_q, h_q)\) and \(\eta_2(x_q, h_q)\) correspond to the occurrence probability of finding an horizontal dipole or a vertical dipole, respectively, at point \((x_q, h_q)\). Their signs depend on the orientation of the dipoles. If the slope of the water table is not too high (say <15°, sin \(\theta\) ≈ 0), the self-potential is approximated by:
\[
\varphi(P) \approx \sum_{q=1}^{Q} I_1(x - x_q, e - h_q) \Gamma_{2,q}
\]
Equation (17) is equivalent to retaining only the vertical contribution of each elementary dipole only. As the intensity of the dipole moment is proportional to the hydraulic difference \(h - h_0\), we define a new distribution,
\[
\alpha(x_0, z_0) = \frac{\eta_2(x_q, z_0)}{(h_q - h_0)}
\]
The \(\alpha(x_q, h_q)\) (expressed in m⁻¹) values are contoured to provide a tomographic image of the location of the water table as isovalue of \(\alpha(x_q, h_q)\) represent potential locations of the water table. To reduce the nonuniqueness of the problem, it is enough to know the water table at a given location (via a piezometer) to determine the water table everywhere assuming that the electrokinetic conversion factor is homogeneous in the investigated area.

[24] The previous approach is relatively similar to that used by Birch [1998] except that we have normalized the cross-correlation integral by \(C_1\). In Figure 4, we have tested the influence of this normalization. The normalization by \(C_1\)
Figure 4. Determination of the position of the water table using the tomographic algorithm discussed in the main text. The thick plain line indicates the position of the water table used in the forward problem (we used a Gaussian distribution in these synthetic cases). (a) Use of the Birch’s algorithm [Birch, 1998]. (b) Use of the model developed in the main text. The isovalues of $\alpha(x_q, h_q)$ (expressed in m$^{-1}$) are contoured.
improves clearly the tomographic algorithm by comparison with that proposed by Birch [1998]. The value of the coupling coefficient can be found by trial-and-error substitution of a series of values \( C_0 \) in order to fit the self-potential profile (a Simplex algorithm can be used for this purpose). A similar algorithm is developed in Appendix B using the electrical field rather than the self-potential distribution at the ground surface. Note that the equations are slightly more complex, but the electrical field, unlike the electrical potential, is free from any additive constant. The use of this algorithm is very rapid (less than 10 s using a desktop computer for all the examples discussed in this paper).

4.3. Use of the Simplex Algorithm

The previous approach is valid under the assumption that the slope of the water table is \(<15^\circ\). We propose in this section the use of another algorithm, namely the Simplex algorithm [Nedler and Mead, 1965; Caceci and Cacheris, 1984], to reconstruct the depth and the shape of the water table avoiding the previous assumption. We assume that either the electrokinetic coupling coefficient \( C_0 \) or the position of the water table at a given location is known to remove nonuniqueness of the solution. The Simplex algorithm is capable of computing the parameters values that best fit a particular set of \( N \) data points given an analytical function with any number of variables and parameters. It is used here to minimize the following cost or residue function,

\[
R \equiv \sum_{i=1}^{N} \omega_i (\varphi_i - \varphi'_i)^2,
\]  

using the least squares criterion and where \( \varphi_i \) represents the measured electrical potential at the ground surface at point \( i \), \( \varphi'_i \) represents the computed value of the electrical potential at the same point using (1) for the forward problem, and \( \omega_i \) represents a statistical weight given to each data point depending for example about the quality of the record at this station determined, for example, by the standard deviation of a series of measurements. We use \( \omega_i = 1 \) in the remaining of the analysis. The parameters that we wish to optimize are the piezometric heads, \( h_q \), the slopes \( q \), and the electrokinetic coupling coefficient \( C' \) arising in equations (8) to (12) and we choose to fix \( h_0 \) to constrain the solution. Note that \( h_q \) and \( q \) are not independent parameters but different tests of the proposed method show that it is better to perform the inversion on both \( h_q \) and \( q \). The best values of these parameters lies at the minimum of the function \( R \). In other words, we optimize the position and the shape of the water table by minimizing the cost function between the measured self-potential profile and that determined in the forward problem using equation (1) (or its discretized version, equation 8). The minimum of this cost function is here determined using the algorithm “Simp” proposed by Caceci and Cacheris [1984]. The a priori model to initiate the inversion scheme is obtained using the semiempirical approach defined in section 4.1.

Computations with synthetic cases are shown in Figure 5. Note that the algorithm is relatively insensitive to the addition of a white noise to the data. Such contamination comes from heterogeneity in the ground [e.g., Weigel, 1987; Revil et al., 2002a]. This is true as long as the strength of this noise remains smaller than the main signal. In practice, it is better to filter first the measurements.
The polarity of the self-potential signal reported the ground surface in response to steady state pumping that a positive electrical self-potential signal is generated at the water table (solid triangles) determined by the water level at one location, in the pumping well for example. Using this information, the water table is determined every-

Nonuniqueness of the problem is removed if one knows the water level at one location, in the pumping well for example. Using this information, the water table is determined everywhere and represented as a bold line in Figure 7b. There is a relatively good agreement between the predictions of our algorithm and the water table delineated from the water level observed in the piezometers. Results from the Simplex algorithm are shown in Figure 8. Using $e - h_0 = 0.5$ m for the depth of the water table in the reference state, the Simplex algorithm yields $C' \approx -14.2 \text{ mV m}^{-1}$, which values ranges in the upper limit of values shown in Figure 3. There is a good agreement between the shape of the water table determined from the Simplex algorithm and that observed in the piezometers.

5.2. Application to Volcanic Data

[31] We apply now the various methods described in section 4 to the data collected by Jackson and Kauahikaua [1987] over a profile >40 km long down the slope of Kilauea volcano (Figure 9). We take the self-potential reference at the sea level. Application of the semipirical relationship yields $\varphi(P) = hK$ using $h_0 = 0$ here (reference level taken at the seafloor). Two boreholes (see locations in Figures 9) are used to constrain the water level estimate. Agreement between the water table determined from the semipirical approach and the self-potential data and the water level determined in the two boreholes is achieved using $K = -1.4 \text{ mV m}^{-1}$ for the entire profile (Figure 9). In addition, we have determined the shape of the water table from the tomographic algorithm described in section 4.2. The results are shown in Figure 10. Results from the Simplex algorithm are shown in Figure 11. The optimized value of the coupling coefficient is $C' = -9.4 \text{ mV m}^{-1}$.

[32] How does the previous values of $C'$ compare with laboratory measurements performed with consolidated volcanic rocks? Jouniaux et al. [2000] have determined from controlled laboratory experiments the electrokinetic coupling coefficient of volcanic rock samples. They used eleven consolidated samples coming from five volcanic deposits of the different evolutionary stages of Mount Pelée volcano. They conclude by stating that the electrokinetic

---

**Figure 6.** Self-potential signals associated with a pumping well. (a) Distribution of the self-potential anomaly. (b) Relationship between the self-potential and the depth of the water table $(e - h)$ at each piezometer. The linear trend yields $K = -3.2 \text{ mV m}^{-1}$. (c) Position of the piezometers, position of the water table (solid triangles) determined by the water level in the boreholes, and position of the water table (solid line) from the semipirical approach derived in the main text. The data are from Bogoslovski and Ogilvy [1973].

Before to invert them, all the 2-D inversion we performed take less than few minutes using a desktop computer. In all the cases tested (including that shown in the next section), we found a very good convergence of the method as long as the sampling of the self-potential is wide enough to avoid truncature effects at the ends of the profile (which is true for all methods). The algorithm can easily handle the presence of topography as shown in section 5.2.
coupling coefficient $C$ ranges from $-25$ to $-406$ mV MPa$^{-1}$ in the field accounting for ionic strength and pH of the natural groundwater at this volcano. This yields $C' = C \rho g$ (Appendix A) in the range $-0.3$ to $-4.1$ mV m$^{-1}$. Therefore the values determined from our analysis are consistent with these experimental values.

5.3. Comparison of the Different Approaches and Future Trends

[33] The semiempirical approach is the fastest of the three methods presented above. However, the coupling coefficient entering the linear relationship between the self-potential signal and the depth of the water table should be considered here as an empirical constant. Its advantage is its simplicity and to the fact that this method can be directly applied to 3-D cases.

[34] The tomographic algorithm discussed in section 4.2 is relatively rapid, but is limited to 2-D cases with small slope of the water table. The Simplex algorithm is the slowest method in terms of CPU time, but it is also the most rigorous method of inversion of the shape and depth of the water table using (1) with no assumptions made regarding the slope of the water table. However, the use of the Simplex algorithm appears maybe not the best way to...
envision 3-D modeling. Next steps will consist in extending the electrography to 3-D cases including electrical resistivity contrasts in the inversion. This would require a robust and efficient inversion algorithm like the one proposed by Tarantola and Valette [1982] and based on generalized least squares minimization criteria and finite element or finite difference methods to address the forward problem.

6. Concluding Statements

Electrokinetic coupling associated with groundwater flow is responsible for the formation of an electrical source current, which in turn generates an electromagnetic signal around the hydraulic source body. The resulting dipolar moment is usually positive in the groundwater flow direction. Spatial or temporal variation of the piezometric head is responsible for the generation of an electric potential distribution through this mechanism. This electrical field can be recorded at the ground surface using a set of nonpolarizable electrodes. The electrical potential recorded at the ground surface amounts usually tens to hundreds of millivolts. The electrokinetic coupling term entering this problem lies generally in the range $-1 \, \text{mV}$ to $-15 \, \text{mV}$ per meter of hydraulic head. Inversion of self-potential anomalies leads to the reconstruction of the water table depth and shape using either a semiempirical approach or a dedicated tomographic algorithm. Nonuniqueness of the problem is removed if one knows the electrokinetic coupling coefficient (from laboratory measurements or groundwater electrical conductivity) or the hydraulic head in a borehole, which can be the pumping well in absence of strong oxidoreduction processes. Many applications of this work can be envisioned concerning the study of the static or dynamic configuration of the water table from the analysis of the self-potential signals. Examples of these applications will be provided in future works.

Appendix A

[36] We consider a water-saturated porous volume $\Omega_0$ (Figure 1), isotropic and possibly inhomogeneous. When fluid flows through a water-saturated porous rock, electric and hydraulic processes are coupled through the following macroscopic constitutive equations operating at the scale of a representative elementary volume [e.g., Ishido and Mizutani, 1981; Pengra et al., 1999; Revil et al., 1999b, and reference therein]:

$$j = \sigma E - \ell \left( \nabla p - \rho g \right),$$  \hspace{1cm} (A1)

$$u = \ell E - \frac{k}{\eta} \left( \nabla p - \rho g \right),$$  \hspace{1cm} (A2)

$$\ell = -\frac{\varepsilon \zeta}{\eta F},$$  \hspace{1cm} (A3)

$$C \equiv \left( \frac{\partial \varphi}{\partial p} \right)_{p=0} = -\ell / \sigma,$$  \hspace{1cm} (A4)

where $j$ is the electrical current density at the scale of the representative porous volume (in A m$^{-2}$), $u$ is the

Figure 8. Determination of the position of the water table using the Simplex algorithm to perform a best fit with the self-potential profile (duration of the computation 22 s, 1000 iterations). Note that the two small negative anomalies relative to infiltration from the drainage ditches have been removed from the self-potential distribution prior to the inversion. The dashed line in Figure 8b represents a smooth of the water table from the data points inverted using the Simplex algorithm and equation (1).

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where $j$ is the electrical current density at the scale of the representative porous volume (in A m$^{-2}$), $u$ is the

Figure 9. Application of the semiempirical approach developed in the main text to the determination of the water table in a volcanic context. There is a good agreement between the water table determined from the self-potential field and the semiempirical equation developed in the main text and that observed in the two available boreholes as shown in the cross section (field data are from Jackson and Kauahikaua [1987]). We use $C = -1.4 \, \text{mV m}^{-1}$ for the entire profile.
volumetric fluid flux (in m s\(^{-1}\)) (Darcy velocity), 
\[ E = \frac{\nabla \varphi - \partial A}{\partial t} \] is the electric field (in V m\(^{-1}\)) (\( \varphi \) is the electrical potential and \( A \) is the magnetic potential vector), \( p \) is the pore fluid pressure (in Pa), \( g \) is the gravity acceleration vector (in m s\(^{-2}\)) and intrinsic permeability (in m\(^2\)) of the porous material, respectively, \( \varepsilon_f \) is the dielectric constant of the groundwater (in F m\(^{-1}\)), \( \rho_f \) and \( \eta_f \) are the density (in kg m\(^{-3}\)) and the dynamic shear viscosity of the groundwater (in Pa s), and \( \ell \) is a coupling term (in m\(^2\)V s\(^{-1}\)) between the generalized Darcy and Ohm’s equations, and \( C \) (in V Pa\(^{-1}\)) is the streaming potential coupling coefficient. The parameter \( \zeta \) (in V) entering into the determination of the coupling coefficient \( \ell \) is the so-called “zeta potential” (see section 3). The term \( F \) is the (dimensionless) electrical formation factor arising in the theoretical description of the electrical conductivity of porous media.

When looking closely at equations (A1) and (A2), it can be shown that the second equation can be safely decoupled from the first equation if the only component of the electrical field is that produced through the electrokinetic coupling [e.g., Sill, 1983; Ishido, 1989; Revil et al., 1999b, section 3.1]. Using this approximation, we recover the classical Darcy equation:

\[ u \approx -\frac{k}{\eta_f} \left( \nabla p - \rho_f g \right) = -K \nabla h. \]  

where \( K \) is here the hydraulic conductivity (in m s\(^{-1}\)), which is related to the intrinsic permeability through \( K = k \rho_g / \eta_f \), and

**Figure 10.** Application of tomographic algorithm developed in the main text to the determination of the water table from the data shown in Figure 7. The two vertical lines with the solid circles at the end correspond to the two boreholes with the position of the piezometric head. The isovalues of \( \sigma(x_p, h_p) \) (expressed in m\(^{-1}\)) are contoured and represent the potential locations of the water table. The black line below the ground surface corresponds to the location of the water table determined using the VES-2 borehole.

**Figure 11.** Application of the Simplex algorithm to the self-potential signal measured on the Kilauea volcano. A best fit of the recorded self-potential signal is performed using the Simplex algorithm to constrain from equation (1) the best shape of the water table. The computation is performed using \( h_0 = 0 \), i.e., the reference station is located at the sea level (computation time \(<2\) min, 500 iterations). The information relative to position of the water level in the two boreholes is not used in the inversion.
and \( h \) is the piezometric head (in m). We consider very simple hydraulic situations below.

[38] We consider now a piecewise uniform conducting space comprising regions \( \Omega_0 \) with constant electrical conductivities \( \sigma_i \) where \( i = 0, 1, 2, 3, \ldots \). The region \( \Omega_0 \), confined by the surface \( \partial \Omega_0 \), represents the porous volume in which fluid flow takes place. Fluid flow is responsible for a dipolar charge separation through electrokinetic coupling (section 3). This yields a volume distribution of the produced electrical field associated with a variation of the hydraulic head, \( \nabla j \_S \) corresponds to the volume density of the distribution of current source in \( \Omega_0 \), and \( \sigma_0 \) is the electrical conductivity of the source volume \( \Omega_0 \) taken here as constant (\( \sigma = \sigma_0 \) only in the source volume and \( j \_S = 0 \) outside \( \Omega_0 \)). The coefficient \( C' \) describes the sensitivity of the produced electrical field associated with a variation of the hydraulic head.

[39] The Maxwell equations in an homogeneous piece of porous material at rest are:

\[
\varepsilon \nabla \cdot \mathbf{E} = \rho, \quad (A9)
\]

\[
\nabla \times \mathbf{E} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0, \quad (A10)
\]

\[
\mathbf{j} = \sigma \mathbf{E} + j \_S, \quad (A11)
\]

\[
\nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}, \quad (A12)
\]

\[
\nabla \cdot \mathbf{H} = 0, \quad (A13)
\]

where \( \rho \) is here the net charge density, \( \varepsilon \) is the dielectric constant of the porous material, \( \mu \) is the magnetic permeability, \( \mathbf{H} \) is the magnetic field \( \mu \mathbf{H} = \nabla \times \mathbf{A} \), and \( j \_S \) is the current density corresponding to the electrokinetic sources. A direct consequence of equations (A9) and (A12) is the continuity equation for the electrical charge:

\[
\nabla \cdot \mathbf{j} = -\frac{\partial \mathbf{P}}{\partial t}. \quad (A14)
\]

For the electrical field, equations (A6) and (A9)–(A13) yield:

\[
-\nabla \times \nabla \times \mathbf{E} - \frac{1}{\eta} \frac{\partial \mathbf{E}}{\partial t} - \mu \frac{\partial j \_S}{\partial t} = \frac{1}{\varepsilon} \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad (A15)
\]

where \( \eta = 1/(\sigma \mu) \) is here the diffusivity of the electromagnetic surges in the low frequency limit and \( c^2 = 1/(\varepsilon \mu) \) where \( c \) the velocity of propagation of the electromagnetic disturbances in the high frequency limit. The critical frequency below which equation (A7) can be safely taken in its diffusive form is roughly \( 10^6 \)–\( 10^{10} \) Hz depending on the electrical conductivity of the ground. In this paper, we consider hydraulic phenomena as the source of the electromagnetic disturbances. The frequency content of hydraulic phenomena is much smaller than the critical frequency discussed above. Therefore the electrical field obeys the following diffusive-type equation,

\[
\nabla^2 \mathbf{E} - \frac{1}{\eta} \frac{\partial \mathbf{E}}{\partial t} - \mu \frac{\partial j \_S}{\partial t} = 0, \quad (A16)
\]

[40] We note \( \tau \) the characteristic time constant for an electromagnetic disturbance to diffuse between two points separated by the distance \( L \). This time constant is given by \( \tau = L^2/\eta \) = \( L^2 \sigma \mu \). Taking \( L = 1000 \text{ m} \), \( \sigma = 0.1 \text{ S m}^{-1} \), and \( \mu \approx 10^{-6} \text{ H m}^{-1} \), we obtain \( \tau = 0.1 \text{ s} \). Consequently, the electromagnetic disturbances diffuse very quickly in the conductive ground. Continuous tomography of the electrical field brings a quasi-instantaneous information about the evolution of the fluid pressure in a porous volume in which fluid flow takes place. This justifies the use of the quasi-static limit. The continuity equation \( \nabla \cdot \mathbf{j} = 0 \), written in the quasi-static limit, combined with (A6) yields,

\[
\nabla \cdot (\sigma \mathbf{E}) = -\nabla \cdot j \_S, \quad (A17)
\]

where \( \mathbf{E} = -\nabla \varphi \) in the quasi-static limit. Equation (A17) follows also from (A16) using the Lorentz gauge and the quasi-static limit. The electrical potential \( \varphi \) obeys the following boundary-value problem in the whole space:

\[
\nabla^2 \varphi = (\nabla \cdot j \_S)/\sigma_0, \text{ in } \Omega_0, \quad (A18)
\]

\[
\nabla^2 \varphi = 0, \text{ in } \Omega_1, \quad (A19)
\]

\[
\sigma_1 \mathbf{n} \cdot \nabla \varphi_1 - \sigma_0 \mathbf{n} \cdot \nabla \varphi_0 = j \_S \cdot \mathbf{n}, \text{ on } \partial \Omega, \quad (A20)
\]

\[
\sigma_1 \mathbf{n} \cdot \nabla \varphi_1 - \sigma_0 \mathbf{n} \cdot \nabla \varphi_0 = 0, \text{ on } \partial \Omega_{ij}, \quad (A21)
\]

\[
\varphi_1 = \varphi_0, \text{ along } \partial \Omega \text{ and } \partial \Omega_{ij}, \quad (A22)
\]

where (A18) corresponds to the Poisson equation with an electrokinetic source term, (A19) represents the Laplace equation for the electrical potential outside the source volume, (A20), (A21), and (A22) represent boundary conditions for the electrical current contribution normal to each interface (which is discontinuous on \( \partial \Omega \) due to the
electrical current source drop on $\partial \Omega$) and the electrical potential, which is continuous through each interface.

[41] We show now that we can recast this boundary-value problem in an equivalent form in which the source term corresponds to a double layer of charge lying along the surface $\partial \Omega$, that is the interface $\partial \Omega$ carries an equivalent dipolar moment of electrical charge. Let us first discuss the properties of the hydraulic head $h$. The scalar function $C'h$ is a twice-differentiable function of position within $\partial \Omega$, zero outside $\partial \Omega$. The continuity equation $\nabla \cdot j = 0$ yields $\nabla \cdot (\sigma \mathbf{E} + j_0) = 0$, and it follows that $\nabla^2 \psi = 0$, in $\Omega_0$, where $\psi \equiv \varphi - C'h$. This type of (pseudo-) potential was introduced by Fitterman [1978] and Still [1983]. The potential $\psi$ is harmonic in the source region $\Omega_0$. This is also true outside the source volume as $\psi = \varphi$ and because $\varphi$ is harmonic, see (A19). Therefore the potential $\psi$ follows the following boundary-value problem:

$$\nabla^2 \psi = 0, \text{ in } \Omega_0, \text{ and } \Omega_1, \quad (A23)$$

$$\psi_j = \psi_j, \text{ along } \partial \Omega_{\Omega}, \quad (A24)$$

$$\sigma_1 \mathbf{n} \cdot \nabla \psi_j = \sigma_0 \mathbf{n} \cdot \nabla \psi_j, \text{ along } \partial \Omega_{\Omega}, \quad (A25)$$

$$\psi_1 - \psi_0 = C'h, \text{ on } \partial \Omega, \quad (A26)$$

$$\sigma_1 \mathbf{n} \cdot \nabla \psi_1 = \sigma_0 \mathbf{n} \cdot \nabla \psi_0, \text{ along } \partial \Omega. \quad (A27)$$

Equation (A23) means that the potential $\psi$ satisfies the Laplace equation everywhere and (A25) is a typical boundary condition for a dipole charge accumulation lying on the interface $\partial \Omega$. Therefore we have shown that the volume distribution of current dipole moment in the source region $\Omega_0$ is equivalent to a dipole layer lying on the boundary of the source body $\partial \Omega$. The potential drop $\psi_1 - \psi_0 = C'h$ corresponds to the dipolar strength of the source. The boundary condition at the ground surface states $\mathbf{n} \cdot \nabla \varphi = 0$ as the atmosphere is insulating, which means that the electrical field is everywhere tangential to the ground surface. Note that outside the source volume, $\psi = \varphi$ and $C'h = 0$. It follows that the double layer potential created by the interface $\partial \Omega$ (the water table) is given at the observation station $P$ located outside the source volume $\Omega_0$ by equation (1) of the main text.

**Appendix B**

[42] We extend here the tomographic algorithm developed in the main text using the electrical field rather than the electrical potential at the ground surface. The electrical field along the local curvilinear coordinate $u$ is given by:

$$E_u = -\frac{\partial \varphi}{\partial u} = -\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial z} \frac{dz}{dx}\right) \frac{dx}{du}. \quad (B1)$$

The expression of the electrical potential at the ground surface, (8), combined with (B1) yields,

$$E_u(P) = \sum_{q=1}^{Q_1} \Gamma_{1q} L_1(x - x_q, z - h_q) \frac{dx}{du} + \sum_{q=1}^{Q_2} \Gamma_{2q} L_2(x - x_q, z - h_q) \frac{dx}{du}, \quad (B2)$$

$$I_1(x - x_q, z - h_q) \equiv \frac{(x - x_q)^2 - (z - h_q)^2}{(x - x_q)^2 + (z - h_q)^2}^2 (dz/du), \quad (B3)$$

$$I_2(x - x_q, z - h_q) \equiv \frac{-2(x - x_q)(z - h_q) + \left[-(x - x_q)^2 + (z - h_q)^2\right] (dz/du)}{(x - x_q)^2 + (z - h_q)^2}^2, \quad (B4)$$

where $P(x, z)$ is the observation point, $M(x_q z_q)$ is the source point, and $I_1$ and $I_2$ represent the two scanning functions determining the two components of the dipolar moment. The density probability of finding a horizontal or a vertical dipole are given respectively by equations similar to equations (13) to (16) of the main text where $\varphi(x, e)$ is replaced by $E_u(x, z(x))$ and $I_1[x - x_q z(x) - h_q]$ and $I_2[x - x_q z(x) - h_q]$ are given by equations (B3) and (B4). As the intensity of the electrokinetic sources is proportional to the piezometric head difference ($h_q - h_0$), we define, like in the main text, a distribution $\alpha(x_q, h_q) \equiv I_1(x_q z(x_q) - h_0)$. The $\alpha(x_q, h_q)$-values are then contoured to provide a tomographic image of the possible locations of the water table.

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