Self-potential modeling from primary flows

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ABSTRACT

This paper discusses a new method for the investigation of self potentials (SP) based on induced current sources. The induced current sources are due to divergences of the convection current which is driven, in turn, by a primary flow, either heat or fluid. As a result of using this approach there is a shift in emphasis toward the vector flow field and its interaction with current cross-coupling structure when compared with the total potential approach of Nourbehecht (1963) which emphasized the primary flow potential and the voltage cross-coupling. This shift in emphasis is advantageous because it is analogous to the actual physical processes. For example, fluid flow in the ground gives rise to drag (convection) currents, and the interaction of the convection currents with the electrical structure gives rise to the electrical potentials (SP). This simple physical picture should aid in developing a better intuitive understanding of the generation of SP effects.

The convective current approach is easily adapted to numerical modeling techniques, as illustrated by its implementation using a two-dimensional (2-D) transmission surface algorithm. When the primary flow is driven by the gradient of a potential, joint modeling of the primary flow and the resultant SP is possible with this algorithm.

Examples of the SP effects generated by point sources of the primary flow in the presence of simple geometrical structures show the diversity of the possible responses. The various responses can be understood in terms of the distributions of the induced current sources caused by the primary flow.

The results from field studies at Red Hill Hot Springs, Utah, are used in an example of the joint modeling of thermal and SP data.

INTRODUCTION

The self-potential (SP) method is based on measurement of naturally occurring potential differences generated mainly by electrochemical, electrokinetic, and thermoelectric sources. The multiplicity of sources can be either an advantage or a disadvantage. On the one hand, a number of phenomena can be studied with the technique and, on the other hand, the possibility of a number of different sources can sometimes be confusing.

There has been a mild resurgence in the use of the SP method in geothermal exploration (Corwin and Hoover, 1979), in the study of earthquake related phenomena (Fitterman, 1978; Corwin and Morrison, 1977), and in engineering applications (Ogilvy et al, 1969; Bogoslovsky and Ogilvy, 1973).

Older methods of interpretation were mostly based on polarized spheres (de Witte, 1948; Yungul, 1950) or line dipole current sources (Meiser, 1962; Paul, 1965). Although these techniques are useful, they provide little information about the nature of the primary sources. Nourbehecht (1963), drawing on the earlier work of Marshall and Madden (1959), discussed the source mechanisms in detail and provided a technique for the solution of coupled flows which incorporated the primary driving potential. His solution is formulated in terms of a total (pseudo) potential composed of the electric potential and the weighted primary source potential (pressure, temperature, concentration). In this formulation, the total (pseudo) potential depends only on the value of the primary potential at the boundaries where there is a change in the coupling parameters, and its value inside the various media is immaterial. Unfortunately, this aspect of the total (pseudo) potential method has sometimes caused neglect of the details of the primary flow, resulting in some calculations for inappropriate models as will be discussed later.

The purpose of this paper is to present an alternative method for the solution of coupled flow problems that explicitly models both the primary flow and the induced secondary electric potentials (joint modeling). Using this technique with a two-dimensional (2-D) algorithm for potential problems provides a new flexibility in the modeling of SP data.

COUPLED FLOWS

The general equation for coupled flows can be written (Onsager, 1931; Marshall and Madden, 1959; Nourbehecht, 1963)

\[ \mathbf{\Gamma}_i = \sum_j L_{ij} \mathbf{X}_j, \]

where the fluxes \( \mathbf{\Gamma}_i \) (charge, matter, heat, etc.) are related to the various forces \( \mathbf{X}_j \) (gradients of electric potential, pressure, temperature, etc.) through the coupling coefficients ("conductivities") \( L_{ij} \). For many practical applications of coupled flows, we are concerned with secondary electric current flows and potentials which are driven by some other primary flow. When the effects of
The secondary electric potentials on the primary flow are small, the primary flow equation is decoupled and the resulting equations are

$$\Gamma_1 = -L_{11} \nabla \xi$$  (2)

and

$$J_{\text{Total}} = \Gamma_2 = -L_{22} \nabla \xi - \sigma \nabla \phi, \quad (3)$$

where $\Gamma_1$ is the primary flow (solution flux, heat flux, etc.), $L_{11}$ is the primary conductivity (permeability, thermal conductivity, etc.), $\xi$ is the primary potential (pressure, temperature, etc.), $J_{\text{Total}}$ is the total electric current, $L_{22}$ is the cross coupling conductance, $\sigma$ is the ordinary electrical conductivity, and $\phi$ is the electric potential. The decoupled primary flow problem [equation (2)] can be solved separately and used in the solution of the electrical flow problem [equation (3)]. As noted in the Introduction, one technique for the solution of equation (3) uses the total (pseudo) potential $\psi$, where $\psi = \phi + L_{22} \xi / \sigma$. The technique used here takes a different approach. Starting with equation (3), note that the first term is a "convection" current driven by the primary flow and the second term is the usual conduction current driven by the gradient of the electric potential. Using this approach, write equation (3) as

$$J_{\text{Total}} = J_{\text{conv}} + J_{\text{cond}}, \quad (4)$$

where

$$J_{\text{conv}} = -L_{22} \nabla \xi, \quad (5)$$

and

$$J_{\text{cond}} = -\sigma \nabla \phi. \quad (6)$$

If no external current sources are imposed and there are no dc conditions (i.e., $\partial \phi / \partial t = 0$), then the total current is divergenceless $(\nabla \cdot J_{\text{Total}} = 0)$ and

$$\nabla \cdot J_{\text{cond}} = -\nabla \cdot J_{\text{conv}} = \nabla \cdot (L_{22} \nabla \xi) = \nabla L_{22} \cdot \nabla \xi + L_{22} \nabla^2 \xi. \quad (7)$$

Thus there are sources (nonzero divergence) of conduction current wherever there are gradients of the cross-coupling coefficient parallel to the primary flow (flow perpendicular to boundaries) or wherever there are external or induced sources of the primary flow.

The sources of the conduction current given by the right-hand side of equation (7) can then be used to determine the resultant electric potential $\psi$, where $\psi = \phi + L_{22} \xi / \sigma$. The technique used here takes a different approach. Starting with equation (3), note that the first term is a "convection" current driven by the primary flow and the second term is the usual conduction current driven by the gradient of the electric potential. Using this approach, write equation (3) as

$$J_{\text{Total}} = J_{\text{conv}} + J_{\text{cond}}.$$

The use of this analogy is somewhat limited by the fact that geometrical applications deal with the boundary condition of zero normal component of the total current flow at the air-earth interface. The analogous boundary condition on normal $B$ is not commonly used in magnetostatic problems. However, since many geophysicists are familiar with magnetostatics, this analogy should provide some intuitive feeling for the form of the induced electrical flows in coupling problems if the slightly different boundary conditions are taken into account.

The use of equation (7) and the nature of the coupling coefficient can be demonstrated for the case of uniform primary flow perpendicular to plane boundaries (Figure 1). At $x = (0, h)$ there are planes of current sources. Using the boundary conditions of no current flow in regions 1 and 3, the solution of this problem gives constant potentials in regions 1 and 3 and a linearly increasing potential in region 2. The constant electric field $E = -\nabla \phi = L_{22} \nabla \xi / \sigma$ in region 2 drives a conduction current that exactly cancels the convection current $L_{22} \nabla \xi$, and the total current is zero everywhere.

This geometry of flow is similar to that used in typical laboratory measurements of coupling coefficients. For example, if the gradients of primary and electric potential (or the potential drops) are measured under the conditions of zero total current, then the ratio

$$-\left(\frac{\nabla \phi}{\nabla \xi}\right) = \frac{L_{22}}{\sigma} = C_{22}$$

defines the voltage coupling coefficient $C_{22}$ in units of volts per unit of primary potential. If, in addition, the conductivity is measured (or the current is measured under the conditions of zero potential), then the current coupling coefficient $L_{22}$ can be determined. For fluid flow, the primary potential is pressure and the...
coefficient \( C_{21} = -\nabla(\phi \nabla P) \) [\( J_{\text{Total}} = 0 \)] is known as the streaming potential coefficient. The alternative measurement \( L_{21} = -\nabla(\phi \nabla P) \) gives the streaming current coefficient. Because the measurement technique is easier, voltage coupling coefficients are more commonly reported in the literature, and they are used in the modeling that follows.

At this point it might be useful to make some comparisons of the total potential and the convection current approaches. Sources for the total potential arise at boundaries where there is a nonzero primary potential and a change in the voltage coupling coefficient \( C_{21} = L_{21}/\sigma \). The sources for the electric potential in the convection current problem [equation (7)] come from regions where there is a divergence of the convection current \( L_{21} \nabla \xi \). Thus the emphasis shifts from the magnitude of the primary potential and the voltage coupling coefficient to gradients of the primary potential and the current coupling coefficient.

There are a number of reasons why the convection current approach might be preferred. The first is that the solution gives directly the real, measurable electric potential and not a combined potential problem. The second is that the source terms depend upon gradients of the primary potential, and these gradients are more directly connected to the physical generation of cross-coupling effects. For example, in the pressure-flow problem, the velocity of the fluid is given by the product of the permeability and the negative gradient of the pressure. On the microscopic scale, it is the velocity of the fluid in the pores that drags along the excess charge in the diffuse layer; this drag current is the convective current. In fact, the cross-coupled fluid-flow problem (electrokinetic effects) can be reformulated in terms of the velocity field with \( -\nabla \xi \) in equation (7) replaced by the velocity vector \( \Gamma \), and \( L_{21} \) replaced by a velocity cross-coupling coefficient given by \( L_{ij}/\kappa \), where \( \kappa \) is the fluid permeability. This reformulation of the problem is particularly attractive since there are fluid-flow problems of interest where the flow is not derivable from the gradient of a pressure, as in thermally driven fluid convection. Thus the convective current approach is applicable for a class of problems which cannot be solved by the total potential approach.

In other cases, approximate solutions to more difficult flow problems may be obtained if care is taken to model the flow in an appropriate fashion. For example, in the case of fluid flow in the vicinity of the water table, it is not possible to model the flow exactly with a simple potential flow solution. However, the gross geometry of the flow can be anticipated and appropriate boundary conditions can be used to give a similar flow pattern (discussed in detail later).

For simple problems with analytic solutions, there are no particular mathematical advantages to using equation (7) compared with using pseudopotentials. However, for more complicated problems requiring numerical techniques, application of equation (7) is straightforward and the solution of the coupled equations (3) requires no more than the solution of another potential problem. The first step is to use an appropriate program to solve the primary potential problem (fluid flow, heat flow, etc.). The second step consists of using the primary potential solution along with a model for the cross-coupling coefficients to calculate the sources for the electric problem from equation (7). The final step uses the current sources along with an electrical model to determine the resultant electric potentials.

This procedure has been implemented with a 2-D transmission surface algorithm for the solution of potential problems.

**NUMERICAL MODELING**

The transmission surface algorithm for the dc potential problem was given by Madden (1971). A brief review is given here starting with the general potential flow equations,

\[
\Gamma = -L \nabla \xi, \quad (8)
\]

and

\[
\nabla \cdot \Gamma = S, \quad (9)
\]

where \( \Gamma \) is the flux, \( \xi \) is the potential, \( S \) is the source, and \( L \) is the "conductivity." When \( L \) is independent of \( y \) (strike direction), equations (8) and (9) can be Fourier (cosine) transformed in the \( y \)-direction giving

\[
-L(x, z) \frac{\partial \xi(x, \lambda, z)}{\partial x} = \Gamma_x(x, \lambda, z), \quad (10)
\]

\[
-L(x, z) \frac{\partial \xi(x, \lambda, z)}{\partial z} = \Gamma_z(x, \lambda, z), \quad (11)
\]

and

\[
\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_z}{\partial z} + \lambda^2 L \xi = S(x, \lambda, z). \quad (12)
\]

The flux and the potential are in general three-dimensional (3-D), depending upon the nature of the source. Equations (10), (11), and (12) can be approximated by a lumped-element rectangular network (Figure 2) and the difference equation at a node is

\[
Y_z(i, j - 1)[\xi(i, j - 1) - \xi(i, j)] + Y_z(i, j)[\xi(i, j + 1) - \xi(i, j)] + Y_z(i, j)[\xi(i, j - 1) - \xi(i, j)] + Y_z(i, j)[\xi(i, j + 1) - \xi(i, j)]
\]

\[
= S(i, j)AxAz, \quad (13)
\]

where

\[
Y_z = L \Delta z/\Delta x, \quad (14)
\]

\[
Y_z = L \Delta x/\Delta z, \quad (15)
\]

\[
Y = \lambda^2 \Delta x \Delta z, \quad (16)
\]

and

\[
i = 1 \ldots, n,
\]
The set of \( n \times m \) node equations can be reorganized into the matrix equation

\[
C \zeta = S
\]  

(17)

where

\[
C = \text{coefficient matrix} \ [n \times m],
\]
\[
\zeta = \text{node potential vector} \ (n \times m),
\]
and

\[
S = \text{node source vector} \ (n \times m).
\]

One method of solution of equation (17) was outlined by Swift (1967), who also provided the code (Swift, personal communication). The final solution in \((x, y, z)\) space was obtained by an inverse Fourier transform of a suite of \(\zeta(\lambda)\) solutions.

The solution of the coupled problem requires three models for the physical properties: the primary flow resistivity \(L_1\), the voltage cross-coupling coefficients \((C_{21}, = \rho L_{21})\), and the electrical resistivity \(\rho\). For each \(\lambda\) value, the primary flow potentials \(\zeta\) are computed for the primary model given a source distribution. From the divergence equation (7), the electrical source terms are calculated at each node. In the lumped circuit, the divergence equation (13) is used with \(L\) replaced by the cross-coupling coefficients \(L_{21}\). In the last step, the electrical source terms calculated in the previous step are used with the resistivity model to determine the electrical potentials \(\phi\).

The solution of these problems is formulated in equation (17) in terms of a flow source which is commonly taken as a point source. Finite-length line source distributions can be computed easily by simple convolution of the point source solution along the source line. In other applications, a specified potential distribution (Dirichlet boundary condition) rather than flow source distributions may be desirable. A simple procedure for accomplishing this, without disturbing the formal solution of equation (17), was outlined in Killpack and Hohmann (1979).

The results of the model calculations are normalized to dimensionless primary potentials \(\zeta\), and dimensionless electric potentials \(V\), defined below:

\[
\zeta_n = \frac{L_{11} a}{l_\zeta},
\]

(18)

\[
V_n = \frac{\phi}{C_{21} \left( \frac{L_{11} a}{l_\zeta} \right)} = \frac{\phi}{C_{21} \left( \frac{\zeta_n}{\zeta} \right)},
\]

(19)

where \(a\) = size scale, length dimension of one model unit, and \(l_\zeta\) = source for primary flow, in units of \(l_\zeta \times \text{area}\). The true potentials \(\zeta\) and \(\phi\) can be obtained by multiplication by the appropriate factor, taking care to use a consistent set of units. One result of modeling with the voltage-coupling coefficient \(C_{21}\) is that the induced current sources are inversely proportional to the resistivity \((L_{21} = C_{21}/\rho)\). Since the voltage is proportional to the current-resistivity product, the resultant model voltages depend only on resistivity ratios. That is, the same potentials will result for all models that differ only by a multiplicative factor in all the model resistivities.

In the models that follow, the distance scales are given in units of \(a\) and the model parameters are given as resistivities and voltage-coupling coefficients, where \(\rho = \text{electrical resistivity}, \rho_T = \text{thermal resistivity}, \rho_H = \text{hydraulic resistivity or impermeability},\) and \(C = \text{voltage coupling coefficient}\). The parameter units, which are unspecified in most of the models, can be any consistent set or, alternatively, they can be considered dimensionless. If the value of a parameter is unspecified, its value is unity.

Surface boundary conditions for the primary problem require careful consideration because the form of the flow near the air-earth interface can have a profound effect on the resultant electric potentials. For temperature problems the appropriate boundary condition is a constant temperature, which is taken as zero. With

**FIG. 3.** Normalized voltage in the vertical plane \((y = 0)\) for a point pressure source in a homogeneous half-space. The boundary condition at the surface is zero normal pressure gradient. The source of unit strength is at \(x = 0, z = 1\) and distances are in units of \(a\).

**FIG. 4.** Normalized voltage in the vertical plane \((y = 0)\) for a point temperature source in a homogeneous half-space. The boundary condition at the surface is zero temperature. The source of unit strength is at \(x = 0, z = 1\) and distances are in units of \(a\).
this boundary condition there is a normal flux of heat at the surface, and there will be induced electrical sources here if the surface medium has a nonzero coupling coefficient.

While it is correct that the excess pressure is zero from the water table up to the surface, uncrtical use of the zero surface pressure boundary condition in potential-flow problems often results in a nonzero normal gradient and therefore a fluid flow at the air-earth interface. As noted in Sill and Johnsg (1979), it is much more important to model the flow geometry which is predominantly horizontal near the surface of the water table. Horizontal fluid flow at the surface in a potential-flow problem requires a zero vertical gradient of the pressure. In the models, zero vertical gradients are produced by giving the air a vanishingly small hydraulic permeability. Zero vertical gradients at a water table below the surface can be modeled with a thin, very low-permeability layer overlying the saturated material at the position of the water table. In effect the modeled flow is confined by impermeable layers rather than having the flow deviated by variations in the height of the water table.

Figures 3 and 4 show the voltage in a vertical $x,z$ plane generated by point sources of pressure and temperature in a homogeneous half-space with $C \neq 0$. As discussed above, the surface boundary condition for pressure problems is zero normal gradient of pressure, and for the thermal problem it is zero temperature at the surface. Comparing these figures, note that the pressure source produces an electrical anomaly at the surface while the temperature source has an equipotential coincident with the surface. In the case of the pressure source, the surface fluid flow is parallel to the air-earth interface and the only induced electrical source is at the pressure source where $\nabla^2 P \neq 0$. For the temperature problem there are induced electrical sources at the temperature source where $\nabla^2 T \neq 0$ and at the surface where there is a normal flux of heat. On the surface, the induced electrical sources at the interface exactly cancel the effects of the source at depth.

These two cases can be solved analytically, and they both have zero total current, i.e., the electrical current exactly cancels the convection current. The primary potentials and flows have the same geometry as the electrical potentials and currents. The analytical solutions can also be used to check the model calculations. Comparisons of results show errors of a few percent for distances from the source greater than one unit, using a model discretization of one-quarter unit.

Both Nourbehecht (1963) and Fitterman (1978) stated that there is no surface anomaly due to a point source of pressure in a homogeneous half-space. The reason for this result is their use of the surface boundary condition of zero pressure. In this case the desire to have the total potential equal to the electric potential led to the use of an inappropriate boundary condition. As seen above, the more appropriate boundary condition is zero normal gradient, not zero pressure, in which case there is a surface anomaly. The models presented in Fitterman (1978, 1979) use the boundary condition of zero primary potential on the surface which is more appropriate for thermal sources even though he proposed them as valid solutions for pressure-flow problems.

Figure 5 shows the effects of pressure source location with respect to a vertical boundary where there is a change in coupling parameters only. For curve 1, with the source to the left of the boundary, note that the positive induced electrical sources at the vertical interface, where there is flow out of the coupling medium, reduce the magnitude of the potential relative to that in a homogeneous half-space (compare with Figure 3). However, the anomaly is symmetric with respect to the source location. When the source is on the boundary (curve 2), the anomaly is reduced to one-half its value in a homogeneous half-space, since the divergence of the flow on the right-hand side of the boundary is into a region of zero coupling. Curve 3 shows a sharp anomaly, centered at the contact due to the negative induced sources at the vertical contact, where there is flow into the coupling medium.

Figure 6 illustrates how changes in the resistivity ratio attenuate or amplify the pressure-induced anomaly with no change in the form. In the case of the amplified effect, $p_2/p_1 = 10$, there are two possibilities. In the first case, let $p_1 = 1$, $p_2 = 10$, and the
current source induced at the point pressure source remains the same as in the homogeneous case with \( \rho_1 = \rho_2 = 1 \). The current flow on the right-hand side is then in a more resistive medium and the magnitude of the potential is increased. In the second case, let \( \rho_1 = 0.1 \) and \( \rho_2 = 1.0 \). Compared to the homogeneous case \( (\rho_1 = 1) \), the induced current source would increase by a factor of ten since \( C = \frac{L}{p} \). The increase in the magnitude of the potential in this case is the same as that produced by the increase in resistivity in the first case. A similar explanation holds for the reduction in amplitude when \( \rho_1 / \rho_2 = 0.1 \).

Figure 7 shows that changes in the “impermeability” \( \rho_\perp \) across a vertical contact change both the amplitude and the form of the anomaly. In the homogeneous case the only induced current source is at the point pressure source where there is a divergence of flow. When the right-hand side is less permeable \( (\rho_\perp < 10) \), there is an increase in the gradient of the pressure in region 2 and a decrease in the gradient in region 1 compared with the homogeneous case. The induced current sources on the vertical boundary are then negative, and they reinforce the negative effects produced by the induced point source at the pressure source. When \( \rho_\perp = 0.1 \), the gradient of the pressure in region 1 increases and it decreases in region 2. In this case the induced sources on the vertical boundary are positive, and they reduce the effects from the induced point source at the pressure source.

A thermal model, similar to the pressure model of Figure 6, is shown in Figure 8. Here, with a homogeneous resistivity structure \( (\rho_1 / \rho_2 = 1) \), the surface anomaly has a symmetrical dipolar form. The positive peak is over the positive induced sources at the air-earth interface, and the negative is to the right of the contact and is due to the \( \nabla^2 T \neq 0 \) source at \( z = 1 \). Changing the resistivity contrast amplifies the anomaly on the resistive side and reduces it on the conductive side. For a resistivity ratio of ten, the asymmetry is so pronounced that the dipolar form is almost obliterated. Figure 9 shows the effects of overburden on the vertical contact model. Even for a homogenous resistivity the overburden produces an asymmetric dipolar form. The reduction of the positive peak is due to the induced sources at the horizontal contact being at a greater depth. Changing the overburden resistivity amplifies or attenuates the form.
Changes in the quarter-space resistivities (Figure 10) in the overburden model also produce large effects. If the coupling medium has a resistivity less than the other quarter-space, the anomalies are essentially monopolar and asymmetric. Changes in the primary flow resistivities can also produce significant effects on the form of the anomaly (Figure 11). Here the anomalies range from monopolar to dipolar as the thermal resistivity of the quarter-spaces is varied. For a thermal resistivity increase, either \( \rho_{r1} \) or \( \rho_{r2} = 10 \), the temperatures and gradients increase compared to the homogeneous case. This causes an increase in the induced current sources at both the point temperature source and on the horizontal overburden interface on the left-hand side. The resultant potential anomaly is then larger than the homogeneous case (Figure 9). When either of the quarter-spaces is more conductive, the temperatures and gradients are smaller than in the homogeneous case, and this leads to smaller induced electrical sources and potentials.

Although the basic pressure anomaly for the quarter-space model is monopolar (Figures 5, 6, and 7), a dipolar anomaly can be produced if the overburden has very large permeability so that there is vertical flow across horizontal boundaries (Figure 12). This flow pattern is then similar to the temperature flow problem in that significant positive electrical sources are induced at the lower overburden interface. However, changes in the quarter-space electrical resistivities can alter this dipolar form.

Monopolar temperature anomalies can be produced with horizontal boundaries (layer over a half-space) as in Figure 13. For the temperature source in position 1, there is a distribution of negative induced electrical sources along the horizontal boundary where there is vertical heat flow into the coupling medium. With the temperature source at position 2, there is only a point source of current on the boundary and the magnitude of the potential is at a maxi-
FIG. 13. Surface voltage (y = 0, z = 0) for a point temperature source and an overburden model, showing the effects of variations in the source location.

For position 3, there is a negative point source of current at the temperature source and a distribution of positive sources along the horizontal boundary where there is vertical heat flow out of the coupling medium. The sign of the anomaly can be changed by making the overburden the stronger cross-coupling medium.

The interactions of point sources with two vertical interfaces are shown in the dike models of Figures 14 and 15. With a point source located in the center of the dike (model 1, Figures 14 and 15), the surface anomaly is symmetrical because the flows and the induced sources are symmetrical with respect to the center of the dike. For the temperature source, the anomaly is positive over the dike, because of the positive induced sources at the air-earth interface, with negative wings on the sides. The negative portions are the result of the larger effect of the divergence of the heat flow from the point source when viewed from the sides. Moving the point temperature source to the left side of the dike (model 2, Figure 14) enhances the negative effect from the divergence of heat flow and produces a dipolar form. A dipolar form is also produced by a sequential increase in the coupling (model 3, Figure 14).

Reversing the contrast in the thermal model 1 of Figure 14 so that \( C = 0 \) in the dike and \( C = 1 \) in the exterior causes a reversal in signs of the anomaly (not shown). The same reversal in contrast in the pressure model, (model 3, Figure 15) causes no change in the sign, but there is a change in the form and a larger change in the magnitude. In this case there are only negative induced sources on the planes at \( x = 0, 1 \) due to the outward flow.

Moving the pressure source to the left side of the dike produces an asymmetric, negative anomaly (model 2, Figure 15). The broad negative to the left is due to the larger effect of the divergence at the source. As the observation point moves toward the right side, the positive induced source on the plane at \( x = 1 \) rapidly cancels the effects of the negative electrical source at the point source of the divergence of the flow.

The results from model 2, Figure 15, indicate that the combination of a positive pressure source at \( x = 0 \) and a negative source at \( x = 1 \), on either side of the dike, would produce a dipolar anomaly. In general, any relatively uniform fluid flow across a dike would tend to produce a dipolar form.

Plan views \( (z = 0) \) of the contoured surface voltage for models

FIG. 14. Surface voltage \( (y = 0, z = 0) \) for a point temperature source and a dike, showing the effects of variations in the source locations and coupling parameters.

FIG. 15. Surface voltage \( (y = 0, z = 0) \) for a point pressure source and a dike, showing the effects of variations in the source locations and coupling parameters.
FIG. 16. Contours of surface voltage ($x$, $y$ plane, $z = 0$) for point sources and a dike. (a) Point temperature source (model 1, Figure 14) (b) Point pressure source (model 1, Figure 15).

1 of Figures 14 and 15 are shown in Figure 16. For a temperature source (Figure 16a), the positive part of the dipolar anomaly is elongated along strike ($y$-direction); for a pressure source (Figure 16b), the monopolar anomaly tends to be elongated perpendicular to strike. The extension of the thermal positive anomaly along strike is easily understood as a result of the localization of the positive induced sources at the air-earth interface along the dike. In the case of the pressure source, the rapid decrease in the anomaly along strike results from cancellation of the negative point source by the distribution of the positive induced sources on the two vertical boundaries.

FIELD EXAMPLE

The Monroe-Red Hill (Utah) geothermal system is an example of deep circulation along a fault zone that has been relatively well studied (Mase et al., 1978). A limited SP survey over the Red Hill area (Figure 17) showed a modest anomaly of dipolar form, with the positive to the southeast of the fault over the altered volcanics and a low to the northwest over the alluvium. Dipole-dipole measurements (Mase et al., 1978) show a low-resistivity zone adjacent to the fault and just to the northwest of the spring. A weak extension of this low-resistivity zone to the northwest is roughly coincident with the potential low of the SP anomaly.

Several factors suggest that a thermoelectric effect might be an appropriate source for the anomaly. First, there is the dipolar form of the anomaly and the general tendency for dipolar forms for thermal anomalies near vertical contacts (Figures 8 and 16a). Second, the position of the high over the volcanics is consistent with the notion that clay alteration in the less porous volcanics would produce a larger thermal cross-coupling coefficient than in the more porous alluvial material.

Figure 18 shows the physical properties model and the location of the thermal sources. The resistivity model is a generalization of models in Mase et al. (1978). The main features are a steeply dipping fault separating the volcanics on the east (201-m) from the alluvium on the west. Away from the fault and near the surface the alluvium is moderately resistive (25–502-m). Near the fault and at depth the alluvium is more conductive, probably because of leakage of thermal waters and alteration. The thermal resistivity contrast between the volcanics and the alluvium is based on average values reported in Mase et al. (1978). The heat source distribution in the model represents the circulation of hot water up the fault and horizontal leakage into the alluvium. The temperature
The calculated and observed temperatures and vertical heat fluxes are in reasonable agreement considering the coarseness of the mesh (25 m) and the fact that several of the observed heat fluxes are based on gradient data at depths less than 10 m. The total heat input into the model is 0.36 MW, which is much less than the total conductive heat loss (1.5 MW) from Red Hill. However, the thermal anomaly at Red Hill is elongated along the direction of the fault. For a 400-m swath normal to the fault (about the width of the SP anomaly), the total conductive heat loss is about 0.5 MW, which is only slightly greater than the heat input to the model. A comparison of the observed and calculated SP anomalies is shown in Figure 19. The comparison is reasonably good, although it should be noted that the observed anomaly is not exactly symmetrical about the centerline. This result was produced by an iterative procedure which involved the adjustment of the cross-coupling parameters until a satisfactory fit was produced. The initial models had a contrast in coupling only at the fault contact, but this configuration did not produce a negative over the alluvium of a magnitude similar to that observed. Subsequent changes in the cross-coupling parameters of the 5Ω-m material below 100 m depth produced additional negative current sources that gave rise

**Properties Model**

<table>
<thead>
<tr>
<th>NW</th>
<th>-600M</th>
<th>-400M</th>
<th>-200M</th>
<th>0</th>
<th>200M</th>
<th>400M</th>
<th>600M</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_T = 0.6)</td>
<td>(C = 0)</td>
<td>(\rho = 50)</td>
<td>(\rho_T = 0.6)</td>
<td>(C = 0)</td>
<td>(\rho = 25)</td>
<td>(\rho_T = 0.6)</td>
<td>(C = 0)</td>
<td>(\rho = 20)</td>
</tr>
<tr>
<td>100M</td>
<td>(\rho_T = 0.6)</td>
<td>(C = 1.27)</td>
<td>(\rho = 5)</td>
<td>(\rho_T = 0.6)</td>
<td>(C = 0.65)</td>
<td>(\rho = 5)</td>
<td>(\rho_T = 0.4)</td>
<td>(C = 2.5)</td>
</tr>
</tbody>
</table>

\[\text{Total source strength} = 0.36 \times 10^6 \text{W}\]

**Thermal Model Temperature Contours in °C**

- **Model Heat Flow**: 1300 mW/M²
- **Observed Heat Flow**: (1200) mW/M²

**Figure 18.** (a) Physical properties used to model the data at Red Hill. (b) Comparison of the observed and calculated temperatures at Red Hill.
The model results presented demonstrate the basic forms of the induced SP response and how the anomalies are changed in form and amplitude by changes in the model parameters. A field example demonstrates the joint modeling of thermal and SP data at Red Hill Hot Spring, Utah. Although the derived cross-coupling coefficients might be considered large, the modeling technique provides a method for testing the constraints on the physical parameters.

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REFERENCES


**SUMMARY**

An alternative method for modeling SP based on induced current sources due to primary flows was presented. The method was implemented using a transmission surface algorithm that provides modeling capabilities for 3-D distributions of sources and 2-D structures.

Fig. 19. Comparison of the observed and modeled SP anomaly at Red Hill. Model properties in Figure 18a.