UNDERSTANDING PHYSICS

SERIES FOR JEE MAIN & ADVANCED

The No. 1 Textbooks of Physics for JEE Main & Advanced by DC Pandey
The Most Reliable Author for JEE Physics
Dear Aspirants,

By now your studies would be in full swing after such a valuable summer break. The spectrum seed which was sown few years back is also taking the shape of a tree, gradually with multiple branches. The overwhelming response given by you plays a vital role in its growth.

The goal of Physics is to achieve a general understanding of physical workings of the entire universe. Many people however see little joy in Physics, might be because they get may stuck out while working on it. Actually the language in which Physics is framed has Mathematics in its roots i.e., to be a perfectionist in Physics a good command over Mathematics is required. However, examples, analogies and experiments are also required to have a perfect insight along with. This subject is so thoroughly embedded in our civilization that it often seems to be part of the fabric of ordinary life. For example, we (all) are subject to Newton’s laws of motion, whether knowingly or unknowingly. We cannot move our body or drive a car or toss a ball in a way that violates his rules. Do you know? Even after giving such a valuable contribution to our society what were his thoughts about Physics

“I do not know what I may appear to the world, but to myself I seem to have been a boy playing on the seashore, and diverting myself I now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay undiscovered before me.”

ISSAC NEWTON

We are trying hard to provide more and more material for your practice within the purview of our limitations. Hope you all are also trying best to Crack the hard nut called EXAMINATION. The spectrum team will be anxiously waiting for your feedbacks and comments.

Good luck

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Tanmay Shekhawat JEE AIR 11th Rank (02)

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Q Achieving top position was a surprise for you or you were expecting it?
Tanmay- It was as expected, I was expecting a top rank.

Q Please share your feelings after achieving such a mile stone?
Tanmay- Feeling very happy and top of the world.

Q In how many attempts did you get this success?
Tanmay- First attempt

Q How did you manage to prepare for board examinations and competitive exams simultaneously?
Tanmay- Put maximum focus on JEE.

Q Please share the secret of your success with our readers.
Tanmay- Regular study, study with full concentration.

Q Tell us something about your source of inspiration.
Tanmay- I had a dream to become an IITian since my childhood and this dream cemented with the time.

Q What was your study plan; please explain in details so that our readers can also follow it?
Tanmay- Put your heart into your studies.

MESSAGE TO FUTURE ASPIRANTS

Study when you are comfortable and put your heart in it. Always focus on your weak areas, Revise as much as possible, practice the previous year papers. Don’t waste your time, don’t take stress on your mind.

SCORE CARD

Board: Physics 93%, Chemistry 94%, Mathematics 99%, Overall 88.6%
Q: Did your Study Plan remain same throughout the year; at the start of the session, after completion of syllabus and just 1-2 months before the exam or you had different plans for different phases?
Tanmay: I had increased the study time between Mains and Advanced and focused much on chemistry in this period.

Q: How many hours of study, is sufficient for such a success?
Tanmay: If anyone is attending Coaching classes, then he/she must study another 3 to 4 hours without any disturbance.

Q: Tell us something about your study techniques like. How do you memorize the facts, how did you have your revision and how you assessed your preparation from time to time.
Tanmay: I use to study with all my heart involved in it, without pressure and studied till my full concentration, I did not study half hearted.

Q: Tell us about your family and the role of your family members in guiding you towards this spectacular performance in the exam?
Tanmay: My Parents and elder brother are very helpful for me but My Mother, who is fighting with cancer, is real source of inspiration.

Q: Share something about the Weak & Strong points of your personality.
Tanmay: Inorganic Chemistry is the weak point and maths is the strong point.

Q: Did you devote equal time to all the subjects or more weightage or one or two subjects as compared to other subjects?
Tanmay: Maximum time to chemistry and equal weightage to Physics and Maths.

Q: Were there any weak areas of yours in any subject(s)? How did you cope up with those?
Tanmay: Inorganic Chemistry, I have given maximum time to this subject. Studied as many books after JEE mains. Books written by Dr. R K GUPTA and J D LEE were very useful for me.

Q: How your teachers/ Mentors helped in achieving this goal?
Tanmay: Dr. R L POONIA, Director Samparpan, always helped me in overcoming the weakness of chemistry, provided good books and given extra time to clear the doubts.

Q: Which is your dream institution and which stream do you wish to pursue?
Tanmay: IIT Mumbai, Computer Science.

Q: Besides studies, what do you do i.e., what are your hobbies and how you match up with these hobbies during your studies? What you do during the breaks from your studies?
Tanmay: I Like Football and Basketball very much but kept myself away from it for 2 years.

Q: Nowadays, Social Media like Facebook, Twitter etc., is a big trend? Do you also engage with these, does it affect the studies? Please give some advice to our readers.
Tanmay: I kept myself away from Social Media for two years.

Q: Please tell us about the books and other reference material that ensured your success?
Tanmay: PHYSICS – D C PANDEY, GALAXY CHEMISTRY – R K GUPTA, J D LEE, BALAJI MATHS - Arihant

Q: Did you find Arihant books helping you to get this success?
Tanmay: R K GUPTA, Inorganic chemistry proved very useful apart from this D C Pandey, physics books were very useful.
Kirchhoff's Law and Measuring Instruments

Kamal Upreti

Physics Cosmos brings a collection of very important topics for JEE Main/PMT exams. Questions from these topics appear regularly in these exams.

If several resistors and cells are connected such that they cannot be reduced to simple series and parallel arrangements, e.g. unbalanced Wheatstone bridge or dissimilar cells connected in parallel or a loop in a circuit containing capacitor, Ohm's law becomes insufficient to solve the problem. To analyse such problems of "Complex DC and AC Circuits", Gustav Kirchhoff in 1882 gave the following two laws:

(A) Kirchhoff's First Law

This law is also known as junction rule or current law (KCL). According to it, the algebraic sum of currents meeting at a junction is zero, i.e. \( \Sigma I = 0 \).

This law implies that current reaching a junction is equal to the current leaving the junction. Treating the current positive if it is reaching a junction and negative if leaving it, in figures (a), (b) and (c), we have

\[
I - I_1 - I_2 = 0 \quad I + I_1 - I_2 = 0 \quad I + I_1 + I_2 = 0
\]

This law is simply a statement of "conservation of charge" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

(B) Kirchhoff's Second Law

This law is also known as loop rule or potential law (KVL) and according to it, "the algebraic sum of the changes in potential in complete closed loop is zero" i.e. \( \Sigma V = 0 \).

While calculating sum of potential drops across each component, an increase of potential is taken positive and a decrease of potential is taken negative (or you can take just opposite).

This law is just another form of "Law of conservation of energy"

If there are \( n \) meshes in a circuit, the number of independent equations in accordance with loop rule will be \( (n-1) \).
PROBLEM SOLVING STRATEGY

APPLYING KIRCHHOFF’S RULES

Process consists of following steps:

**Step I** Assume currents $I_1, I_2, \ldots$ etc., in different branches of circuits. Current distribution must follow KCL i.e. current going out of any component must equal to the current going into that component.

**Step II** Select closed loops in given circuit, number of loops selected = number of unknown currents ($I_1, I_2, \ldots$).

**Step III** Assign ‘+’ and ‘−’ signs to ends of each resistor and also to the terminals of cell. To assign sign to a resistor, remember that current flows from ‘+’ to ‘−’ end of resistor. Cell is assigned ‘+’ and ‘−’ as per polarity of terminals. For example, consider given circuit

![Circuit Diagram](image)

Now, we assign (assume) currents as shown

![Current Diagram](image)

Now, we assign ‘+’ and ‘−’ signs

![Sign Diagram](image)

**Step IV** Move in a loop clockwise or anti-clockwise and add all potential drops. Note that $\Sigma V = 0$ by KVL in any closed loop. In our example, we take loop ABCDA anti-clockwise. Now, potential drop in branch $AB = -I_1 R_1$ (when we move from $A$ to $B$ sign of potential which occurs later is ‘−’ across $R_1$).

**Note** Take sign occurs ‘later’ while adding potential drop across any resistor or cell.

So, we have in loop ABCDA, $-I_1 R_1 - I_2 R_4 - I_3 R_5 + V_1 = 0$

Similarly form $\Sigma V = 0$ for other loops selected.

**Step V** Now solve these equations to get value of unknowns $I_1, I_2, \ldots$ etc.

**Note** If value of any current obtained is negative that means that current is flowing in opposite direction to that of assumed one.

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**Worked Out Problem**

**Example 1** Calculate the current through each resistance in the given circuit as shown in figure.

![Circuit Diagram](image)

Also, calculate the potential difference between the points $B$ and $E$. Use

$E_1 = 6 \text{ V}$  \hspace{1cm} $E_2 = 8 \text{ V}$  \hspace{1cm} $E_3 = 10 \text{ V}$

$R_1 = 5 \Omega$  \hspace{1cm} $R_2 = 10 \Omega$  \hspace{1cm} $R_3 = 4 \Omega$

Assume that all the cells have no internal resistance.

**Sol.** We proceed as:

**Step I** Assume unknowns $(x, y, \ldots)$ for currents in different branches of the circuit. Use Kirchhoff’s current law at the junctions so that the number of unknowns introduced is minimum.

Suppose $x$ be the current through $R_1$ and $y$ be the current through $R_2$ as shown in figure.

Kirchhoff’s current law at the junction $B$ gives a current $(x - y)$ through $R_2$.

![Current Diagram](image)

**Step II** Now, we select loops $ABEFA$ and $BCDEB$. Only two loops are selected as we have to determine only two variables $x$ and $y$.

**Step III** Assign signs ‘+’ and ‘−’ to ends of resistors as per direction of current through them.

Also, assign signs to terminals of cells as per their polarities.

![Sign Diagram](image)

**Step IV** Calculate sum of potential drops in each loop and equate it to zero, ($\Sigma V = 0$). Each potential drop must be provided with the sign that occurs later across each component (resistor or cell) of circuit.

In loop $ABEFA$ (we are going anti-clockwise from $A$)

- For cell of emf $E_1$, potential drop $= -E_1$ (negative plate occurs later).
For resistance $R_3$, potential drop = current $\times$ resistance

$$= yR_3$$ (the sign occurs later)

For cell $E_3$, potential drop

$$= + E_3$$ (check sign)

For resistance $R_i$, potential drop = $xR_i$ (check sign)

So, finally we get the sum $-E_1 + yR_3 + E_3 + xR_i = 0$

With values equations will be

$$-6 + y4 + 10 + x5 = 0$$

$$5x + 4y = -4 \quad \ldots (i)$$

And similarly for loop BCDEB (we go clockwise from point $B$)

$$-(x - y)R_2 - E_2 + yR_3 + E_3 = 0$$

$$-x - y10 - 8 + y4 + 10 = 0$$

$$14y - 10x = -2$$

$$7y - 5x = -1 \quad \ldots (ii)$$

**Step V** Now we solve Eqs. (i) and (ii), we get

$$11y = -5 \Rightarrow y = -\frac{5}{11}$$

Putting the value of $y$ in Eq. (i), we get

$$5x + 4\left(-\frac{5}{11}\right) = -4 \Rightarrow x = \frac{20 - 44}{11} = -\frac{24}{55}$$

And

$$x - y = \frac{24}{55} - \left(-\frac{5}{11}\right) = \frac{1}{55}$$

**Step VI** A negative value of current means opposite to assumed direction. So, our currents flow as circuit shown below.

Now we calculate potential difference between $B$ and $E$.

The potential difference between any two points in a circuit is calculated by adding the changes in potential while going through any path from one point to the other point. Hence, it goes from $E$ to $B$ through $R_3$.

$$V_B - V_E = +yR_3 + E_3 = \left(-\frac{5}{11}\right)4 + 10 = \frac{90}{11} V$$

**Example**

Twelve identical resistors are connected to form edges of a cube. Find the equivalent resistance between the corners marked 1 and 2 as shown in figure below.

**Step I** Suppose a current $i$ enter the point $A$ and leave through the point $B$ due to battery of $V$ volt connected across them.

**Step II** By symmetry we assume currents $i_1$, $i_2$, and $i_3$ going in through point $A$. Further currents are distributed as shown in figure.

**Step III** From KCL, we have

$$i = i_1 + 2i_2 \ldots (i)$$

Applying KVL for the loop (1 → 4 → 3 → 2 → 1), we have

$$-i_2R - i_3R_i - i_2R + i_2R = 0 \quad \text{or} \quad i_1 = i_2 + i_3 \ldots (ii)$$

Applying KVL for the loop (4 → 3 → 6 → 5 → 4), we have

$$-i_3R + (i_2 - i_3)R + 2(i_2 - i_3)R + (i_2 - i_3)R = 0$$

$$4i_2 - 5i_3 = 0 \ldots (iii)$$

On solving Eqs. (i), (ii), (iii), we get

$$i_1 = \frac{7}{12}, i_2 = \frac{5}{24}, i_3 = \frac{1}{6}$$

**Step IV** The potential difference across

$$(1 \rightarrow 2)$$ is

$$i_1 \times R = \frac{7}{12} \times R$$

Thus, $iR_{eq} = \frac{7}{12} R$

or

$$R_{eq} = \frac{7}{12}$$

**Measuring Instruments**

In DC circuits we need to know the value of current or voltage or resistances. Few of our measuring instruments are

(A) **Galvanometer**

It is a deflection instrument which shows a deflection when a current flows through its coil. The main parts of a galvanometer are as shown in figure below.
It is worth nothing that
(a) In case of moving coil galvanometer, deflection is
directly proportional to the current that passes
through it.
i.e. \( I \propto 0 \) or \( I = K0 \)
where, \( K \) is called galvanometer constant.
(b) The current required for full scale deflection in a
galvanometer is called full scale deflection current
and is represented by \( I_g \) (\( \sim \) mA).
(c) The deflection per unit current is called current
sensitivity of the galvanometer i.e. \( \frac{\theta}{I} = \frac{1}{K} \)
(d) The reciprocal of current sensitivity i.e. current per
unit deflection is called figure of merit.
i.e. \( FM = \frac{I}{\theta} = \frac{1}{Si} = K \)

(B) Ammeter
To measure current, a galvanometer must be connected in
series in the circuit but due to large resistance of its coil, it
is not possible to use it directly.

To convert a galvanometer into an ammeter of a certain
range say \( I \), a small resistance \( S \) (called shunt) is
connected in parallel with the galvanometer so that the
current passing through the galvanometer of resistance \( G \)
becomes equal to its full scale deflection value \( I_g \) as shown
in figure above. This is possible only if
\[
I_g \cdot G = (I - I_g)S
\]
i.e. \[
S = \frac{I_g \cdot G}{I - I_g}
\]

(C) Voltmeter
It is a device used to measure potential difference and is
always put in parallel with the circuit element across which
potential difference is to be measured as shown in the
figure. Voltmeter \( V_1 \) will measure potential difference
across resistance \( R_1 \), \( V_2 \) across resistance \( R_2 \) and \( V \) across
\( (R_1 + R_2) \) with \( V = V_1 + V_2 \).

To convert a galvanometer into a voltmeter of certain
range say \( V \), a high resistance \( R \) is connected in series
with the galvanometer so that current passing through the
galvanometer of resistance \( G \) becomes equal to its full scale deflection value \( I_g \)
as shown in figure alongside.
i.e. \( V = I_g (R + G) \)
or \( R = \frac{V}{I_g} - G \)

(D) Wheatstone Bridge
A Wheatstone bridge is an arrangement of four resistances which can be
used to measure one of the
unknown value among them in terms of the rest.
The bridge is said to be
balanced when deflection in galvanometer is zero i.e. \( I_g \) is zero and hence \( V_B = V_D \) so that
\[
(V_A - V_B) = (V_A - V_D)
\]
and \( (V_B - V_C) = (V_D - V_C) \)
i.e. \( I_1 P = I_2 R \)
and \( I_1 Q = I_2 S \)
or \( \frac{P}{Q} = \frac{R}{S} \)

Eq. (i) is the required result under condition of balance for
a Wheatstone bridge.

(E) Meter Bridge
A meter bridge is a "user-friendly" form of Wheatstone
bridge, where two of resistances are replaced by a
resistance wire. In case of a meter bridge, the resistance
wire AC is 100 cm long. In balance condition, there is no
current through the galvanometer.
If balance occurs at point B,
\[
AB = l, \quad BC = (100 - l)
\]
then \( \frac{S}{R} = \frac{(100 - l)}{l} \)

Unknown resistance, \( S = \frac{(100 - l)}{l} R \)
(F) Potentiometer

Potentiometer consists of a long resistive wire made up of manganin or constantan.

(i) The specific resistance ($\rho$) of potentiometer wire must be high but its temperature coefficient or resistance ($\alpha$) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to jockey.

(iii) The EMF of driver cell must be greater than the value of unknown potential difference to be balanced against potential difference of balance length.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the diameter of potentiometer wire must be uniform everywhere.

Potential gradient (X) Potential difference (or fall in potential) per unit length of wire is called potential gradient.

By trial, balance point 'J' where galvanometer shows no deflection can be found. In balance condition, potential drop across $V$ length $AJ$ is equals to emf $E$ of the cell.

If $V = E$, then no current will flow in galvanometer circuit this condition is known as null deflection position, length $l$ is known as balancing length.

In balanced condition,

$$ E = Xl \text{ or } V_{AJ} = Xl = \frac{V}{L} \text{ or } l = \frac{iR_{AB}}{L} $$

or,

$$ E = \left( \frac{e}{L} \right) \frac{R_{AB}}{L} \times l $$

Sensitivity of potentiometer A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(i) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(ii) To increase the sensitivity of potentiometer

(a) The resistance $R_n$ in primary circuit will have to be increased.

(b) The length $L$ of potentiometer wire will have to be increased.

| Difference between Voltmeter and Potentiometer |
|-----------------|-----------------|
| **Voltmeter**   | **Potentiometer** |
| (i) Its resistance is high but finite. | Its resistance is high but infinite. |
| (ii) It draws some current from source of emf. | It does not draw any current from the source of known emf in balance condition. |
| (iii) The potential difference measured by it is lesser than the actual potential difference. | The potential difference measured by it is equal to actual potential difference. |
| (iv) Its sensitivity is low. | Its sensitivity is high. |

Applications of Potentiometer

(i) To determine the internal resistance of a primary cell

Initially in secondary circuit key $K'$ remains open and balancing length ($l_1$) is obtained. Since, cell $E$ is in open circuit so its emf balances on length $l_1$ i.e.

$$ E = Xl_1 \quad \ldots (i) $$

Now key $K'$ is closed so cell $E$ comes in closed circuit. If the process is repeated again, then potential difference $V$ balances on length $l_2$ i.e.

$$ V = Xl_2 \quad \ldots (ii) $$

By using formula internal resistance, $r = \left( \frac{E}{V} - 1 \right) R$

(ii) Comparison of emf’s of two cells

Let $l_1$ and $l_2$ be the balancing lengths with the cells $E_1$ and $E_2$ respectively, then $E_1 = Xl_1$ and $E_2 = Xl_2 \Rightarrow E_1 \neq E_2 \neq \frac{l_1}{l_2}$
Example 3
While finding internal resistance of a cell by using a potentiometer, a student of class XII made following observations:

(i) When cell is shunted by 5 Ω resistor, balance length of potentiometer wire is 2 m.
(ii) When cell is shunted by 10 Ω resistor, balance length is 3 m.

Find internal resistance of cell used in above experiment.

Sol. For any cell,

\[ E - ir = iR \]

So,

\[ i = \frac{E}{(r + R)} \]

and

\[ V = iR = \left( \frac{E}{r + R} \right) R \]

In case (i),

\[ V = i_1 x l = \left( \frac{E}{5 + r} \right) R \]

⇒

\[ 2x l = \left( \frac{E}{5 + r} \right) \cdot 5 \quad \ldots (i) \]

In case (ii),

\[ 3x l = \left( \frac{E}{10 + r} \right) \cdot 10 \quad \ldots (ii) \]

On dividing Eqs. (i) and (ii), we get

\[ \frac{2}{3} = \left( \frac{10 + r}{10} \right) \times \left( \frac{5}{5 + r} \right) \]

So,

\[ r = 10 \ \Omega \]

Example 4

(i) In given circuit, find the potential difference between the points A and B.

(ii) Now we wish to measure this potential difference by using a voltmeter of resistance 2 kΩ. Find the reading of the voltmeter and percentage error.

(iii) Solve part (ii) if the voltmeter were of resistance 20 kΩ.

\[
\begin{array}{c}
\text{A} \\
\downarrow 200 \ \Omega \\
\downarrow \\
\downarrow 200 \ \Omega \\
\uparrow 40 \ \text{V}
\end{array}
\]

What conclusion do you draw from the results you get in the above parts?

Sol.

**Step I**
As both the resistances are same, 40 V will be divided equally among both the resistances. Hence, the potential difference across A and B is 20 V.

**Step II**
Equivalent resistance of 200 Ω and 2 kΩ is

\[ R_1 = \frac{200 \times 2000}{200 + 2000} = \frac{2000}{11} \ \Omega \]

Example 5

An electrical circuit is shown in figure. Calculate the potential difference across the resistor of 400 Ω, as will be measured by the voltmeter V of resistance 400 Ω.

\[
\begin{array}{c}
\text{V} \\
\downarrow \\
\downarrow \\
\downarrow 10 \ \text{V}
\end{array}
\]

* \[ I_1 \]

\[ R_1 = 100 \ \Omega \\
R_2 = 100 \ \Omega \\
R_3 = 400 \ \Omega \\
R_4 = 200 \ \Omega \\
I_1 = 1 \ \text{A}
\]

**Sol.**

**Step I**
Applying Kirchhoff’s law in loop JMGDJ, we get

\[
-\frac{1}{2} x 400 + (I_1 + I_2 - I_3) 200 + (I_2 - I_3) x 100 = 0
\]

\[-500I_2 + 200I_1 + 300I_2 = 0
\]

⇒

\[ 2I_1 + 3I_2 - 5I_3 = 0 \quad \ldots (i) \]

**Step II**
Applying Kirchhoff’s law in CDEFBC, we get

\[-100I_2 - 100I_1 - 100I_3 = 0
\]

⇒

\[ I_1 - 2I_2 + I_3 = 0 \quad \ldots (ii) \]
Multiplying Eq. (ii) by 2 and subtracting from Eq. (i), we have

\[ 2l_1 + 3l_2 - 5l_2 - 2l_1 + 4l_2 - 2l_2 = 0 \]

\[ \Rightarrow \quad l_2 = l_3 \]  

\[ \quad \ldots (iii) \]

**Step III** Applying Kirchhoff’s law in ABFEGHA, we get

\[-3l_1 - 2l_2 + 2l_3 + 0.1 = 0 \]  

\[ \quad \ldots (iv) \]

Multiplying Eq. (ii) by 3 and adding it to Eq. (iv), we get

\[-3l_1 - 6l_2 + 3l_3 - 2l_2 + 2l_3 - 3l_1 + 0.1 = 0 \]

\[ \Rightarrow \quad -8l_2 + 5l_3 + 0.1 = 0 \]

\[ \Rightarrow \quad -8l_2 + 5l_3 + 0.1 = 0 \]

**Step IV** Therefore, potential difference across JM,

\[ V = \frac{0.1}{2} \times 400 = \frac{0.1}{2} \times 400 = \frac{20}{3} = 6.67 \text{ V} \]

**ALTERNATIVE METHOD**

We can redraw the circuit as shown in figure. The equivalent resistance between G and D is

\[ R_{GD} = \frac{400 \times 400}{400 + 400} = 200 \Omega \]

\[ \Rightarrow \quad \frac{R_{GD}}{R_{GB}} = \frac{R_{GR}}{R_{GD}} \]

Therefore, it is the case of a balanced Wheatstone bridge.

The equivalent resistance across G and B is

\[ R_{GB} = \frac{300 \times 300}{300 + 300} = 150 \Omega \]

Therefore, current is

\[ i = \frac{V}{R_{GB}} = \frac{10}{15} = \frac{1}{1.5} \text{ A} \]

Since, the current is divided at G into two equal parts. The currents i/2 is farther divided into two equal parts at M. Therefore, the potential difference across the voltmeter is

\[ V = \frac{i}{4} \times 400 = \frac{1}{4} \times 100 = \frac{20}{3} \text{ V} \]

**Smart Practice**

**Single Correct Answer Type**

1. Calculate the energy stored in the capacitor of capacitance 2µF. The voltmeter gives a reading of 15 V and the ammeter A reads 15 mA.

\[ \text{(a) } 5 \mu \text{J} \quad \text{(b) } 10 \mu \text{J} \quad \text{(c) } 0.5 \mu \text{J} \quad \text{(d) } 0 \mu \text{J} \]

2. In the given circuit, \( R_1 \neq R_2 \) and the reading of the voltmeter is the same, irrespective of whether the switch \( S \) is open or closed. Then, which of the following is correct?

\[ \text{(a) } I_{11} = I_{1} \quad \text{(b) } I_{11} = I_{12} \quad \text{(c) } I_{11} = I_{1} \quad \text{(d) None of these} \]

3. In a potentiometer, an auxiliary cell of 2 V is connected along with a rheostat at 5 Ω, then balance length obtained for a cell of 1.5 V is around

\[ \text{(a) } 1 \text{ cm} \quad \text{(b) } 100 \text{ cm} \quad \text{(c) } 1000 \text{ cm} \quad \text{(d) } 1.5 \text{ m} \]

4. In the circuit shown in the figure, XY is a potentiometer wire of 100 cm long. The circuit is connected up as shown with switches \( S_1 \) and \( S_2 \) open, a balance point is found at Z. After switch \( S_1 \) has remained closed for some time, it is found that
contact at Z must be moved towards Y to maintain a balance.

Which of the following is the most likely reason for this?
(a) The cell $V_1$ is running down
(b) The cell $V_2$ is running down
(c) The wire $XZ$ is getting warm and its resistance is increasing
(d) The resistance $R_1$ is getting warm and increasing in value

5. In a practical Wheatstone bridge circuit, when one more resistance of 100 $\Omega$ is connected in parallel with unknown resistance $X$, then the ratio $l_1/l_2$ becomes 2. $l_1$ is the balance length, $AB$ is a uniform wire. Then, the value of $X$ must be

(a) 50 $\Omega$
(b) 100 $\Omega$
(c) 200 $\Omega$
(d) 400 $\Omega$

6. In the given figure, points $A$ and $B$ are connected by a perfectly conducting wire. Calculate the current through $AB$.

(a) 2 A
(b) 1 A
(c) 1.5 A
(d) 2.5 A

7. The two ideal batteries having emf $E_1$ and $E_2$ are connected as shown in the figure. The values of resistances are chosen in such a way that ammeter reading is zero. The reading of voltmeter will be (consider the meters to be ideal)

(a) $10 \pi \sin \pi t$ cm s$^{-1}$
(b) $10 \pi \cos \pi t$ cm s$^{-1}$
(c) $20 \pi \sin \pi t$ cm s$^{-1}$
(d) $20 \pi \cos \pi t$ cm s$^{-1}$
11. If the switch at point P is opened (shown in the figure), choose the correct option.

(a) The current in \( R_1 \) would not change
(b) The potential difference between point X and the ground would increase
(c) The current provided by the battery would increase
(d) The emf provided by the battery (assumed to have no internal resistance) would change

12. Consider the circuit shown in the figure. Find the charge on capacitor C between A and D in steady state.

(a) \( C \epsilon \)  
(b) \( C \epsilon / 2 \)  
(c) \( C \epsilon / 3 \)  
(d) Zero

13. If \( R_1 = 2R_2 = 4R_3 = 8R_4 = 16R_5 \), find the resistance in which maximum heat is generated, if key K is closed.

(a) \( R_1 \)  
(b) \( R_2 \)  
(c) \( R_4 \)  
(d) \( R_5 \)

14. In the following circuit, the resistance of potentiometer wire is 100 \( \Omega \). Power consumption in potentiometer wire is same when jockey is placed at 10 cm from end A or end B. Internal resistance of cell (r) is

(a) 30 \( \Omega \)  
(b) 6 \( \Omega \)  
(c) 60 \( \Omega \)  
(d) 3 \( \Omega \)

15. When an ammeter of negligible internal resistance is inserted in series with circuit, it reads 1A. When the voltmeter of very large resistance is connected across X, it reads 1 V. When the points A and B are shorted by a conducting wire, the voltmeter measures 10V. The internal resistance of the battery (r) is equal to

(a) zero  
(b) 0.5 \( \Omega \)  
(c) 0.2 \( \Omega \)  
(d) 0.1 \( \Omega \)

16. The figure shown gives values of \( R_1 \) and \( R_2 \), the balance point for jockey is at 30 cm from A. When \( R_3 \) is shunted by a resistance of 10\( \Omega \), jockey has to slide 20 cm to obtain balance point. The values of \( R_1 \) and \( R_2 \) are

(a) \( \frac{10}{5}, \frac{5}{5} \)  
(b) \( \frac{40}{3}, \frac{40}{7} \)  
(c) \( \frac{40}{7}, \frac{40}{3} \)  
(d) None of these

17. In the circuit as shown, \( V_A = V_B \). The resistance of each wire AB, AC, etc., are shown in the figure. Find the current in AB.

(a) \( V/2R \)  
(b) \( 4V/11R \)  
(c) \( V/6R \)  
(d) \( V/3R \)

18. Find out the value of current through 2\( \Omega \) resistance for the given circuit.

(a) 5 A  
(b) 2 A  
(c) zero  
(d) 4 A
19. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. Find the current in the circuit, so that the ammeter shows maximum deflection
(a) 100.1 mA  
(b) 1000.1 mA  
(c) 10.01 mA  
(d) 1.01 mA

20. R₁, R₂ and R₃ are different values of R, A, B and C are the null points obtained corresponding to R₁, R₂ and R₃, respectively. For which resistor, the value of X will be the most accurate and why?

(a) R  
(b) 3R  
(c) 2R  
(d) 4R

21. A resistance of 2 Ω is connected across one gap of a meter bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω connected across the other gap when resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is
(a) 3 Ω  
(b) 4 Ω  
(c) 5 Ω  
(d) 6 Ω

22. A meter bridge set up as shown below:

Galvanometer shows null deflection at 52 cm mark end corrections for ends A and B are 1 cm and 2 cm respectively. Value of existence X observed in experiment will be
(a) 10.2 Ω  
(b) 10.6 Ω  
(c) 10.8 Ω  
(d) 11.1 Ω

23. During an experiment with a meter bridge, the galvanometer shows a null point, when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the scale used in the meter bridge is 1 mm. The unknown resistance is
Answers with Explanation

1. (d) From the given figure,

\[ V_A - V_B = 15 \text{ V} \quad \text{...(i)} \]

and \[ V_C - V_D = \frac{E}{R_{\text{total}}} \]

From Eqs. (i) and (ii), we get

\[ V_A - V_D = 15 \times 10^{-3} \times (1 + 999) = 15 \text{ V} \]

\[ V_C - V_D = (15 - 15) = 0 \]

Therefore, energy stored in the capacitor is zero.

2. (a) This is a case of balanced Wheatstone bridge.

3. (c) \( V_{AB} = \) potential difference across ends of wire

\[ I_{AB}R_{AB} = \frac{E}{R_{\text{total}}} \times R_{AB} = \frac{2}{20} \times 15 = 1.5 \text{ V} \]

So, potential gradient of potentiometer wire

\[ V_{AB} = \frac{1.5}{1000} = 1.5 \times 10^{-3} \text{ V/cm} \]

\[ \therefore \text{ Potential of } 1.5 \text{ V is balanced against a length} \]

\[ l' = \frac{1.5}{1.5 \times 10^{-3}} = 1000 \text{ cm} \]

4. (a) If \( V_A \) decreases, then the potential difference across \( XZ \) will decrease. So, to have the same potential difference, i.e. \( V_Z \) across \( XZ \), length across \( XZ \) has to be increased or \( Z \) should be moved towards \( Y \).

5. (b) The Wheatstone bridge is in balanced condition, so

\[ 100 \times \frac{L_1}{100 + x} = \frac{L_2}{L_2} \]

\[ \therefore \frac{L_1}{L_2} = 2 \text{ or } x = 100 \Omega \]

6. (b) Using Kirchhoff’s Voltage Law (KVL), we get

\[ 2I_1 + (I - I_1) = 0 \text{ or } I = 3I_1 \]

and \[ (I - I_1) + 2I_1 = 4 \]

Solving, we get

\[ I_1 = 1 \text{ A} \text{ and } I = 3 \text{ A} \]

Therefore, current through \( AB = (3 - 2)A = 1 \text{ A} \)

7. (b) The ammeter is not showing any reading. It means the potential drop across \( C_b \) due to \( E_1 \) is equal to that of the emf \( E_2 \). And this is the reading of the voltmeter.

8. (d) For balanced meter bridge (null deflection), we get

\[ \frac{55}{R} = \frac{20}{80} \text{ or } R = 220 \Omega \]

9. (a) Potential difference across AC is zero.

10. (d) \[ V_P = \frac{20 + x}{40} \]

\[ V_0 = 2 \]

\[ \Rightarrow V_P - V_0 = \frac{x}{10} \]

\[ 2 \sin \pi t = \frac{x}{10} \]

or \[ x = 20 \sin \pi t \]

Hence, \[ \frac{dx}{dt} = (20 \pi \cos \pi t) \text{ cm s}^{-1} \]

11. (b) If \( P \) is disconnected, \( R_{\text{eq}} \) of circuit increases, hence less current is drawn.

Therefore, as \( i \) decreases, \( V_e = E - iR_1 \) increases.

12. (d) In steady state, there will be no current in any branch. So, \( A \) and \( D \) will be at the same potential. Hence, charge on capacitor between \( A \) and \( D \) will be zero.

13. (d) All the resistances are in parallel. The potential difference across each resistance is same.

\[ \Delta H = i^2R = \frac{V^2}{R} \text{ or } \Delta H = \frac{1}{R} \]

Therefore, \( R_e \) is the smallest and generates maximum heat.

14. (a) Case I As 10 cm from A, power dissipation,

\[ R_e = \left( \frac{E}{r + 10} \right)^2 \times 10 \]

Case II As 10 cm from B, power consumption

\[ P_e = \left( \frac{E}{r + 90} \right)^2 \times 90 \]

As,

\[ R_1 = \frac{P_2}{P_1} \]

\[ r = 30 \Omega \]
15. (c) As, \( i = 1\)A and \( r \) is the internal resistance of the battery.

\[
12 = (x + y + r) (i) \quad \text{...(i)}
\]

Also,

\[
1 = (x)(1) \quad \text{or} \quad x = 1 \Omega
\]

Voltage across \( x \) (when \( A \) and \( B \) are shorted)

\[
10 = \left( \frac{12}{x + r} \right) x
\]

or

\[
10 = \left( \frac{12}{1 + r} \right) \quad r = \frac{6}{5} \Omega \quad \therefore \quad \text{or} \quad x = \frac{1}{5} \Omega = 0.2 \Omega
\]

16. (a) \( \frac{R_1}{R_2} = \frac{30}{70} = \frac{3}{7} \) \( \text{...(i)} \)

\[
\frac{R_1}{R_2} = 1
\]

or

\[
R_1 = \frac{R_2 \times 10}{R_2 + 10} = \frac{3}{7} R_2 \quad \text{...(ii)}
\]

or

\[
R_2 = \frac{40}{3} \Omega \quad \text{or} \quad R_1 = \frac{40}{7} \Omega
\]

17. (d)

\[
V = IR_3
\]

Hence, current through resistance is \( V/3R \).

18. (a) By applying Kirchhoff’s rules, we conclude that current in each loop remains confined to the loop itself.

\[
\therefore \quad \text{Current through} \ 2 \ \Omega \ \text{resistor is zero.}
\]

19. (a) \( V_{ab} = i_g \cdot G = (i - i_g) S \Rightarrow i = \left( 1 + \frac{G}{S} \right) i_g \)

Substituting the values, we get \( i = 100.1 \text{mA} \)

20. (a) Slide wire bridge is most sensitive when the resistance of the four arms of bridge is same.

So, value of \( X \) is most accurate when, \( X = R \).

21. (a) \( R > 2 \Omega \)

\[
\begin{align*}
100 - x &> x \\
\therefore \quad P &> \frac{R}{s} \\
\text{We have,} & \\
\frac{2}{R} &> \frac{x}{100 - x} \\
\frac{R}{2} &> \frac{x + 20}{80 - x}
\end{align*}
\]

On solving Eqs. (i) and (ii), we get, \( R = 3 \Omega \)

22. (b) Using the concept of balanced Wheatstone bridge, we have

\[
\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{X}{(52 + 1)} = \frac{10}{(48 + 2)}
\]

\[
\therefore \quad X = 10 \times 53 \times \frac{50}{50} = 10.6 \Omega
\]

23. (c) \( R = \frac{x}{100 - x} \times 90 = \frac{40 \times 90}{100 - 40} = 3600 \Rightarrow R = 60 \Omega
\)

\[
\frac{dR}{R} = 100 \times \frac{x}{(100 - x)} \ dx
\]

\[
\therefore \quad dR = 100 \times \frac{0.1 \times 60}{40 \times 60} = 0.25 \Omega
\]

The unknown resistance = \( (60 \pm 0.25) \Omega \)

24. (5) \( i_g (G + 4990) = V \)

\[
\Rightarrow \frac{6}{1000} (G + 4990) = 30
\]

\[
\Rightarrow G + 4990 = \frac{30 \times 1000}{6} = 5000
\]

\[
\Rightarrow G = 10 \Omega
\]

\[
V_{ab} = V_{ac}
\]

\[
\Rightarrow i_g (G + 15 - i_g) S
\]

\[
= \frac{6}{1000} \times 10 = \left( \frac{15}{1000} - \frac{6}{1000} \right) S \Rightarrow S = \frac{60}{1494} = \frac{2n}{249}
\]

\[
= \frac{249 \times 30}{1494} = \frac{2490}{498} = 5
\]

25. (b) To increase range of ammeter, a parallel resistance is required, which is given by

\[
S = \left( \frac{i_g}{i_g - i_g} \right) G \Rightarrow S = \left( \frac{50 \times 10^{-6}}{5 \times 10^{-6} - 5 \times 10^{-6}} \right) \times 100 = 1 \Omega
\]

To convert microammeter into a voltmeter, a high series resistance is used

\[
R = \frac{V_{ac}}{i_g} \Rightarrow R = \frac{10}{50 \times 10^{-6} - 100} \Rightarrow R = 200 \times 10^6 \Omega
\]
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EXAM CRUX

- In Physics paper, 21 questions were asked from Class XII and 19 questions from Class XI. Most of the questions were asked from all chapters but more weightage given to modern Physics.
- According to topicwise distribution, the number of questions were asked from Class XI:Unit and Dimensions (1), Kinematics (3), Laws of Motion (3), Work, Power and Energy (3), Centre of Mass, Impulse and Momentum (1), Rotation (1), Gravitation (1), SHM (2), Solids and Fluids (1), Heat and Thermodynamics (3).
- As per topicwise distribution in Class XII: Optics (4), Current Electricity (3), Electrostatics (2), Magnetics (3), Electromagnetic Induction and AC (2), Modern Physics (7).
- As per the difficulty level around 50% questions are easy, 39% are relatively moderate and rest 11% are tough.

by Arihant Team

1. Electron microscope is based on the principle of [Dual Nature (Easy)]
   (a) photoelectric effect
   (b) wave nature of electron
   (c) superconductivity
   (d) laws of electromagnetic induction

2. Force is given by the expression, 
   \[ F = A \cos(Bx) + C \cos(Dt) \], where x is displacement and t is time. The dimension of \( D/B \) is same as that of [Unit and Dimensions (Moderate)]
   (a) velocity
   (b) velocity gradient
   (c) angular velocity
   (d) angular momentum

3. A car accelerates from rest with 2 m/s² on a straight line path and then comes to rest after applying brakes. Total distance travelled by the car is 100 m in 20 seconds. Then, the maximum velocity attained by the car is [Motion in 1 D (Easy)]
   (a) 10 m/s  
   (b) 20 m/s  
   (c) 15 m/s  
   (d) 5 m/s

4. A body is falling freely from a point A at a certain height from the ground and passes through points B, C and D (vertically as shown below) so that BC = CD. The time taken by the particle to move from B to C is 2 s and from C to D is 1 s. Time taken to move from A to B in seconds is [Motion in 1 D (Tough)]
   (a) 0.6  
   (b) 0.5  
   (c) 0.2  
   (d) 0.4

5. A particle moves from (1, 0, 3) to the point (−3, 4, 5) when a force \( \mathbf{F} = \mathbf{i} + 5\mathbf{k} \) acts on it. Amount of work done in joules is [Work, Power and Energy (Easy)]
   (a) 14  
   (b) 10  
   (c) 6  
   (d) 15

6. A particle is projected with velocity \( 2\sqrt{gh} \) and at an angle 60° to the horizontal so that it just clears two walls of equal height h which are at a distance 2h from each other. The time taken by the particle to travel between these two walls is [Motion 2D (Tough)]
   (a) \( 2\sqrt{\frac{2h}{g}} \)  
   (b) \( \frac{h}{\sqrt{2g}} \)  
   (c) \( 2\sqrt{\frac{h}{g}} \)  
   (d) \( \frac{h}{g} \)

7. A body of mass 20 kg is moving on a rough horizontal plane. A block of mass 3 kg is connected to the 20 kg mass by a string of negligible mass through a smooth pulley as shown in the below figure. The tension in the string is 27N. The
8. Two masses \( m_1 \) and \( m_2 \) are placed on a smooth horizontal surface and are connected by a string of negligible mass. A horizontal force \( F \) is applied on the mass \( m_2 \) as shown in the figure. The tension in the string is \[ \frac{m_1}{m_1 + m_2} F \]

9. A body of mass 3 kg moving with a velocity \( (2\hat{i} + 3\hat{j} + 3\hat{k}) \) m/s collides with another body of mass 4 kg moving with a velocity \( (3\hat{i} + 2\hat{j} - 3\hat{k}) \) m/s. The two bodies stick together after collision. The velocity of the composite body is \[ \frac{1}{7}(4\hat{i} + 5\hat{j} - 3\hat{k}) \]

10. A simple pendulum of length \( L \) carries a bob of mass \( m \). When the bob is at its lowest position, it is given that the minimum horizontal speed necessary for it to move in a vertical circle about the point of suspension. When the string is horizontal, the net force on the bob is \[ \sqrt{10} \text{ mg} \]

11. A system of two particles is having masses \( m_1 \) and \( m_2 \). If the particle of mass \( m_1 \) is pushed towards the centre of mass of particles through a distance \( d \), by what distance the particle of mass \( m_2 \) should be moved, so as to keep the centre of mass of particles at the original position? \[ \frac{m_1}{m_1 + m_2} d \]

12. A thin uniform circular disc of mass \( M \) and radius \( R \) is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity \( \omega \). Another disc of same thickness and radius but of mass \( \frac{1}{8} M \) is placed gently on the first disc coaxially. The angular velocity of the system is now \[ \frac{8}{9} \omega \]

13. A 9 kg solution is poured into a glass U-tube as shown in the figure alongside. The tube’s inner diameter is \( 2\sqrt{\pi} \) m and the solution oscillates freely up and down about its position of equilibrium (\( x = 0 \)). The period of oscillation in seconds is \( 1 \text{ m}^3 \) of solution has a mass \( \mu = 900 \text{ kg} \), \( g = 10 \text{ m/s}^2 \). Ignore frictional and surface tension effects.

14. Two bodies of masses 100 kg and 8100 kg are held at a distance of 1 m. The gravitational field at a point on the line joining them is zero. The gravitational potential at that point in J/kg is \( G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{ kg}^2 \)

15. An elastic spring of unstretched length \( L \) and force constant \( k \) is stretched by a small length \( x \). It is further stretched by another small length \( y \). Work done during the second stretching is \[ \frac{k}{2} (x + 2y) \]

16. A soap bubble of radius 1.0 cm is formed inside another soap bubble of radius 2.0 cm. The radius of an another soap bubble which has the same pressure difference as that between the inside of the smaller and outside of large soap bubble, in metres is \[ 6.67 \times 10^{-3} \]

17. A slab of stone of area 3600 cm² and thickness 10 cm is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on upper surface of the slab. In one hour, 4.8 kg of ice is melted. The thermal conductivity of the stone in Js⁻¹m⁻¹K⁻¹ is (Latent heat of ice = 3.36 \times 10^5 \text{ J/kg})

18. The surface of a black body is at a temperature 727°C and its cross-section is 1 m². Heat radiated from this surface in one minute in joules is (Stefan’s constant = 5.7 × 10⁻⁸ W/m² K⁴) [Heat: Transfer of Heat (Easy)]
   (a) 34.2 × 10⁵ (b) 2.5 × 10⁶ (c) 3.42 × 10⁵ (d) 2.5 × 10⁶

19. Two moles of a gas is expanded to double its volume by two different processes. One is isobaric and the other is isothermal. If W₁ and W₂ are the works done respectively, then [Thermodynamics (Easy)]
   (a) W₂ = \frac{W₁}{\log_2 2}  
   (b) W₂ = W₁  
   (c) W₂ = W₁ \log_2 2  
   (d) W₂ = W₁ \log_{10} 2

20. Uranium has two isotopes of masses 235 and 238 units. If both of them are present in uranium hexafluoride gas, find the percentage ratio of difference in rms velocities of two isotopes to the rms velocity of heavier isotope. [KTG (Moderate)]
   (a) 1.64  
   (b) 0.064  
   (c) 0.64  
   (d) 6.4

21. A source of frequency 340 Hz is kept above a vertical cylindrical tube closed at lower end. The length of the tube is 120 cm. Water is slowly poured in just enough to produce resonance. Then, the minimum height (velocity of sound = 340 m/s) of the water level in the tube for that resonance is [Oscillations and Waves (Moderate)]
   (a) 0.75 m  
   (b) 0.25 m  
   (c) 0.95 m  
   (d) 0.45 m

22. A thin convex lens of focal length f made of crown glass is immersed in a liquid of refractive index n₁ (n₁ > nₐ), where nₐ is the refractive index of the crown glass. The convex lens now is [Ray Optics (Easy)]
   (a) a convex lens of longer focal length  
   (b) a convex lens of shorter focal length  
   (c) a divergent lens  
   (d) a convex lens of focal length (n₁ - nₐ)f

23. Two convex lenses of focal lengths f₁ and f₂ form images with magnifications m₁ and m₂, when used individually for an object kept at the same distance from the lenses. Then, f₁ / f₂ is [Ray Optics (Moderate)]
   (a) \frac{m₁(1 - m₂)}{m₂(1 - m₁)}  
   (b) \frac{m₁(1 - m₂)}{m₂(1 - m₁)}  
   (c) \frac{m₂(1 - m₁)}{m₁(1 - m₂)}  
   (d) \frac{m₂(1 - m₁)}{m₁(1 - m₂)}

24. With the help of a telescope that has an objective of diameter 200 cm, it is proved that light of wavelengths of the order of 6400 Å coming from a star can be easily resolved. Then, the limit of resolution is [Wave Optics (Easy)]
   (a) 39 × 10⁻⁸ deg  
   (b) 39 × 10⁻⁸ rad  
   (c) 19.5 × 10⁻⁸ rad  
   (d) 19.5 × 10⁻⁸ deg

25. Two charged identical metal spheres A and B repel each other with a force of 3 × 10⁻⁵ N. Another identical uncharged sphere C is touched with sphere A and then it is placed mid-way between A and B. Then, the magnitude of net force on C is [ES (Easy)]
   (a) 1 × 10⁻⁵ N  
   (b) 3 × 10⁻⁵ N  
   (c) 2 × 10⁻⁵ N  
   (d) 5 × 10⁻⁵ N

26. The electrostatic potential inside a charged sphere is given as V = Ar² + B, where r is the distance from the centre of the sphere, A and B are constants. Then, the charge density in the sphere is [ES (Moderate)]
   (a) 16 Aε₀  
   (b) -6 Aε₀  
   (c) 20 Aε₀  
   (d) -15 Aε₀

27. Three unequal resistances are connected in parallel. Two of these resistances are in the ratio 1:2. The equivalent resistance of these three resistors connected in parallel is 1 Ω. What is the highest resistance value among these three resistances if no resistance is fractional? [CF (Moderate)]
   (a) 10 Ω  
   (b) 8 Ω  
   (c) 15 Ω  
   (d) 6 Ω

28. Two electric resistors have equal values of resistance R. Each can be operated with a power of 320 watt W at 220 volt. If the two resistors are connected in series to a 110 volt electric supply, then the power generated in each resistor is [CF (Moderate)]
   (a) 90 watt  
   (b) 80 watt  
   (c) 60 watt  
   (d) 20 watt

29. A current of 1 A is flowing along the sides of an equilateral triangle of side 4.5 × 10⁻² m. The magnetic field at the centroid of the triangle is (μ₀ = 4π × 10⁻⁷ H/m) [Magnetic Effect of Current (Moderate)]
   (a) 4 × 10⁻⁵ T  
   (b) 2 × 10⁻⁵ T  
   (c) 4 × 10⁻⁵ T  
   (d) 2 × 10⁻⁴ T

30. A charged particle (charge = q; mass = m) is rotating in a circle of radius R with uniform speed v. Ratio of its magnetic moment (\(\mu\)) to the angular momentum (I) is [Magnetism and Matter (Easy)]
   (a) \frac{q}{2m}  
   (b) \frac{q}{m}  
   (c) \frac{q}{4m}  
   (d) \frac{2q}{m}

31. Two small magnets have their masses and lengths in the ratio 1:2. The maximum torques experienced by them in a uniform magnetic field are the same. For small oscillations, the ratio of their time periods is [Magnetism and Matter (Easy)]
   (a) \frac{1}{2\sqrt{2}}  
   (b) \frac{1}{\sqrt{2}}  
   (c) \frac{1}{2}  
   (d) 2\sqrt{2}

32. Two coils have mutual inductance 0.005 H. The current changes in the first coil according to equation I = I₀ sin ωt, where I₀ = 10 A and ω = 100π rad s⁻¹. The maximum value of induced emf in the second coil is [EML (Moderate)]
   (a) 5  
   (b) 5π  
   (c) 0.5π  
   (d) π
33. A capacitance of \( \left( \frac{10^{-3}}{2\pi} \right) \) F and an inductance of \( \left( \frac{100}{\pi} \right) \) mH and a resistance of 10 \( \Omega \) are connected in series with an AC voltage source of 220 V, 50 Hz. The phase angle of the circuit is \( \text{[AC (Moderate)]} \) (a) 60° (b) 30° (c) 45° (d) 90°

34. Two equations are given below:
(A) \( \int \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \)
(B) \( \int \mathbf{B} \cdot d\mathbf{A} = 0 \)

They are \( \text{[EMW (Easy)]} \)
(a) (A) Ampere's law (B) Gauss law for electricity
(b) (A) Gauss law for electric fields (B) Gauss law for magnetic fields
(c) (A) Faraday's law (B) Gauss law for electric fields
(d) Both (A) and (B) represents Faraday's law

35. A charged particle is accelerated from rest through a certain potential difference. The de-Broglie wavelength is \( \lambda_1 \) when it is accelerated through \( V_1 \) and is \( \lambda_2 \) when accelerated through \( V_2 \). The ratio \( \lambda_1 / \lambda_2 \) is \( \text{[Dual Nature (Easy)]} \)
(a) \( V_1^{\frac{1}{2}} : V_2^{\frac{1}{2}} \)
(b) \( V_1^{\frac{3}{2}} : V_2^{\frac{1}{2}} \)
(c) \( V_1^{\frac{1}{2}} : V_2^{\frac{1}{2}} \)
(d) \( V_1^{\frac{1}{2}} : V_2^{\frac{3}{2}} \)

36. If the first line of Lyman series has a wavelength 12154 Å, the first line of Balmer series is approximately \( \text{[Atoms (Moderate)]} \)
(a) 4864 Å (b) 1025.5 Å (c) 6563 Å (d) 6400 Å

37. A certain radioactive element disintegrates with a decay constant of \( 7.9 \times 10^{-10} \) /s. At a given instant of time, if the activity of the sample is equal to \( 5.53 \times 10^{13} \) disintegration/second, then number of nuclei at that instant of time is \( \text{[Nucleus (Moderate)]} \)
(a) \( 7.0 \times 10^{21} \)
(b) \( 4.27 \times 10^{13} \)
(c) \( 4.27 \times 10^{3} \)
(d) \( 6 \times 10^{23} \)

38. The change in current through a junction diode is 12 mA when the forward bias voltage is changed by 0.6 V. The dynamic resistance is \( \text{[Semiconductor Devices (Easy)]} \)
(a) 500 \( \Omega \)
(b) 300 \( \Omega \)
(c) 150 \( \Omega \)
(d) 250 \( \Omega \)

39. A semiconductor has equal electron and hole concentration of \( 2 \times 10^{18} \) m\(^{-3}\). On doping with a certain impurity, the electron concentration increases to \( 4 \times 10^{19} \) m\(^{-3}\), then the new hole concentration of the semiconductor is \( \text{[Semiconductor (Moderate)]} \)
(a) \( 10^6 \) m\(^{-3}\)
(b) \( 10^8 \) m\(^{-3}\)
(c) \( 10^{10} \) m\(^{-3}\)
(d) \( 10^{12} \) m\(^{-3}\)

40. A message signal of 12 kHz and peak voltage 20 V is used to modulate a carrier wave frequency 12 MHz and peak voltage 30 V. Then, the modulation index is \( \text{[Communication (Easy)]} \)
(a) 0.32 (b) 6.7 (c) 0.67 (d) 67

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**Answers with Explanation**

1. (b) Electron microscope is based on the principle of wave nature of electron.

2. (a) **Key Idea** The principle of homogeneity of dimension states that a physical quantity equation will be dimensionally correct, if the dimensions of all the terms occurring on both sides of the equation are same.

   From dimension logic, the quantities of same dimensions can be added or subtracted, so

   \[
   \left[ \frac{\text{L}}{\text{T}^4} \right] = \left[ \frac{\text{L}}{\text{T}} \right] + \left[ \frac{\text{L}}{\text{T}} \right] \Rightarrow \text{Dimension of } \frac{B}{B} = [\text{L}^1 \text{T}^{-1}]
   \]

   \( [\text{L}^1 \text{T}^{-1}] \) is the dimension of velocity.

3. (a) **Given**, Acceleration of car (\( a \)) = 2 m/s\(^2\)

   Distance covered by car in 20 s = 100 m

   We know that, distance travelled by car,

   \[
   s = \frac{1}{2} \frac{a\beta}{\alpha + \beta} t^2
   \]

   \[
   100 = \frac{1}{2} \times \frac{2\beta}{(2 + \beta)} \times 20 \times 20
   \]

   \[
   100 = \frac{\beta}{(2 + \beta)} \Rightarrow \frac{1}{4} = \frac{\beta}{2 + \beta}
   \]

   \[
   \Rightarrow 4\beta = 2 + \beta
   \]

   \[
   \Rightarrow 3\beta = 2
   \]

   \[
   \Rightarrow \text{retardation } \frac{\beta}{\alpha + \beta} = \frac{2 \text{ m/s}^2}{3}
   \]

   \[
   \therefore \text{Maximum velocity (v}_{\text{max}}\) = \frac{a\beta}{\alpha + \beta}
   \]

   \[
   = \frac{2 \times 3}{3 + 20} \times \frac{80}{3} \times \frac{3}{5} = \frac{80 \times 3}{8} = 10 \text{ m/s}
   \]

4. (b) Let velocity of the particle at point B be v.

   Let velocity of the particle at point B be v.

   \[
   BC = 2v + \frac{1}{2} g \times (2)^2
   \]
or \[ BC = 2v + 2g \]

Similarly, \[ 2BC = 3v + \frac{9g}{2} \]

\[ \Rightarrow 2BC = 3v + \frac{9g}{2} \]

From Eq. (i),

\[ 2(2v + 2g) = 3v + \frac{9g}{2} \]

\[ \Rightarrow 4v + 4g = 3v + \frac{9g}{2} \]

\[ \Rightarrow 4v - 3v = \frac{9g}{2} - 4g \]

\[ \Rightarrow v = \frac{g}{2} \] \hspace{1cm} \ldots (ii)

Time taken to move, from point A to B,

\[ v = u + at \Rightarrow \frac{g}{2} = 0 + g \times t \]

\[ \Rightarrow t = \frac{1}{2} \text{ s} \Rightarrow t = 0.5 \text{ s} \]

5. (c) Starting point of particle = \( \hat{i} + 3\hat{k} \)

Ending point of particle = \(-3\hat{i} + 4\hat{j} + 5\hat{k}\) force (F) = \( \hat{i} + 5\hat{k} \)

\[ \therefore \text{Displacement of the particle} \]

\[ \mathbf{s} = (-3\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 3\hat{k}) \]

\[ = -4\hat{i} + 4\hat{j} + 2\hat{k} \]

We know that,

Work done, \( W = \mathbf{F} \cdot \mathbf{s} \)

or \[ W = (\hat{i} + 5\hat{k}) \cdot (-4\hat{i} + 4\hat{j} + 2\hat{k}) \]

or \[ W = -4 + 10 \text{ or } W = 6 \text{ J} \]

6. (a) Given, velocity of particle = \( 2\sqrt{gh} \)

Angle of projection (\( \theta \)) = 60^\circ

Distance covered by particle = \( 2h \)

Now, \[ 2h = 2\sqrt{gh} \cos 60^\circ \]

\[ \Rightarrow \sqrt{h} = \frac{2h}{2} \sqrt{g} \]

Time taken by the particle

\[ \therefore \text{Time taken by the particle} \]

\[ t = 2 \frac{\sqrt{h}}{\sqrt{g}} \]

7. (b)

\[ \text{Given, tension in string } (T) = 27 \text{ N} \]

Mass \( (m) = 3 \text{ kg} \)

Let acceleration of the block be \( a \).

\[ 3g - T = ma \]

\[ \Rightarrow 10 \times 3 - 27 = 3a \]

\[ \Rightarrow 30 - 27 = 3a \]

\[ \Rightarrow a = \frac{3}{3} = 1 \text{ m/s}^2 \]

For the body, we have,

\[ 27 - \mu \times 20g = 20a \]

\[ \Rightarrow 27 - \mu \times 20 \times 10 = 20 \times 1 \]

\[ \Rightarrow 27 - 200\mu = 20 \Rightarrow 200 \mu = 27 - 20 \]

\[ \therefore \text{Coefficient of kinetic friction,} \]

\[ \mu = \frac{7}{200} = 0.035 \]

Coefficient of kinetic friction is 0.035.

8. (a)

According to the question, force, \( (F) \) is applied on \( m_2 \) mass.

So, tension in the string,

\[ T = m_1 \times a \]

\[ \text{or } T = \left( \frac{m_1}{m_1 + m_2} \right) \cdot F \]

\[ \therefore a = \frac{F}{m_1 + m_2} \]

9. (b) Key Idea

In the absence of an external force, the total momentum of the system remains constant or conserved and does not change with time.

\[ \text{i.e. } F_{\text{ext}} = 0 \]

Given, mass of first body \( (m_1) = 3 \text{ kg} \)

Velocity of first body \( (v_1) = (2\hat{i} + 3\hat{j} + 3\hat{k}) \text{ m/s} \)

Mass of second body \( (m_2) = 4 \text{ kg} \)

Velocity of second body \( (v_2) = (3\hat{i} + 2\hat{j} - 3\hat{k}) \text{ m/s} \)

According to law of conservation of linear momentum,

\[ m_1v_1 + m_2v_2 = (m_1 + m_2)v \]

\[ \Rightarrow (2\hat{i} + 3\hat{j} + 3\hat{k}) \cdot 3 + (3\hat{i} + 2\hat{j} - 3\hat{k}) \cdot 4 = (4 + 3)v \]

\[ \Rightarrow 6\hat{i} + 9\hat{j} + 9\hat{k} + 12\hat{i} + 8\hat{j} - 12\hat{k} = 7v \]

\[ \Rightarrow 18\hat{i} + 17\hat{j} - 3\hat{k} = 7v \]

Velocity of the composite body,

\[ v = \frac{1}{7} (18\hat{i} + 17\hat{j} - 3\hat{k}) \]

10. (a) Key Idea

The bob is moving in a vertical circle, so the velocity at lowest point \( v = \sqrt{5rg} \), where \( r \) is radius of a circular path and \( g \) is acceleration due to gravity.

Now, applying conservation of mechanical energy, we get

\[ \frac{1}{2} \times m \times 5g = \frac{1}{2} \times m \times v^2 + mg \]

\[ \text{or } \frac{5}{2} \times g = \frac{v^2}{2} + \text{ or } \frac{3}{2} \times g = \frac{v^2}{2} \]

\[ \text{or } v^2 = 3rg \] \hspace{1cm} \ldots (i)

Now, horizontal force on the bob, \( F_x = F_z = \frac{m \times v^2}{r} \)

\[ = \frac{m \times 3rg}{r} = 3mg \text{ spherical path } \]

Vertical force on the bob, \( F_y = mg \)

\[ \therefore F_{\text{net}} = \sqrt{F_x^2 + F_y^2} \]

\[ = \sqrt{(3mg)^2 + (mg)^2} = \sqrt{10} \text{ mg} \]
11. (c) In two particles system, if the first particle is pushed towards the centre of mass through a distance \( d \). Then, to keep the centre of mass at the same position, the second particle should be moved by the distance \( \frac{m_1}{m_2} d \).

12. (a) Given, mass of first circular disc \( m_1 = M \)
Radius of first circular disc \( r_1 = R \)
Mass of second circular disc \( m_2 = \frac{1}{8} M \)

For first disc,
\[ l_1 = \frac{MR^2}{2} \]  \( \cdots \)(i)
\[ \Rightarrow L_1 = \frac{MR^2 \omega_1}{2} \]  \( \cdots \)(ii)

For first and second discs system,
\[ l_2 = \frac{MR^2}{2} + \frac{M}{8} \frac{R^2}{2} \]
or
\[ l_2 = \frac{9MR^2}{8x} \]  \( \cdots \)(iii)
\[ \Rightarrow L_2 = \frac{9MR^2}{8x} \omega_2 \]  \( \cdots \)(iv)

Now from conservation of angular momentum,
\[ L_1 = L_2 \]
\[ \Rightarrow \frac{MR^2 \omega_1}{2} = \frac{9MR^2}{8x} \omega_2 \Rightarrow \omega_1 = \frac{9}{8} \omega_2 \]

Given,
\[ \omega_2 = \omega \]
\[ \Rightarrow \omega_2 = \frac{8}{9} \omega_1 = \frac{8}{9} \omega \]

13. (a) Diameter of tube = \( \frac{\pi}{5} \) m
\[ \Rightarrow \text{Radius of tube} (r) = \frac{\pi}{5} m \]

Mass of solution \( (\mu) = 9 \text{ kg} \)
We know that,
\[ \mu = \frac{\rho V}{L} \]
or
\[ 9 = 900 \times \pi \times \left( \frac{\pi}{5} \right)^2 \times L \]
or
\[ 9 = 900 \times \frac{\pi^2}{5} \times L \]
or
\[ L = \frac{5}{100 \pi^2} \]  \( \cdots \)(i)

Now,
\[ T = 2\pi \sqrt{\frac{L}{2g}} \]

From Eq. (i),
Time period of oscillation,
\[ T = 2\pi \sqrt{\frac{5}{200 \pi^2 \times 10}} \]
or
\[ T = 2 \times \sqrt{\frac{1}{400}} = \frac{2}{20} \text{ or } T = 0.1 \text{ s} \]

14. (a) Mass of first body \( (m_1) = 100 \text{ kg} \)
Mass of second body \( (m_2) = 8100 \text{ kg} \)
Distance between two bodies = 1m
Distance of null point from 100 kg
\[ x = \frac{d}{\sqrt{\frac{m_1}{m_2} + 1}} \text{ or } x = \frac{1}{\sqrt{\frac{8100}{100} + 1}} \]
or
\[ x = \frac{1}{9 + 1} = \frac{1}{10} = 0.1 \text{ m} \]

The gravitational potential at that point is given by
\[ = -G \frac{m_1 + m_2}{0.1} \]
\[ = -6.67 \times 10^{-11} \left( \frac{100}{9} \right) \left( \frac{8100}{9 \times 10^1} \right) \]
\[ = -6.67 \times 10^{-11} \left( 10000 + 9000 \right) \]
\[ = -6.67 \times 10^{-7} \text{ J/kg} \]

15. (d) Consider an elastic spring of unstretched length \( L \) and force constant \( k \) is stretched by a small length \( x \).

In the first condition,
\[ W_1 = \frac{1}{2} kx^2 \]  \( \cdots \)(i)

In the second condition,
\[ W_2 = \frac{1}{2} k(x + y)^2 \]  \( \cdots \)(ii)

Work done for \( x \) to \( y \) expansion
\[ = \frac{1}{2} k(x + y)^2 - \frac{1}{2} kx^2 \]
\[ = \frac{1}{2} k[x^2 + y^2 + 2xy - x^2] \]
\[ = \frac{1}{2} k[y^2 + 2xy] = \frac{1}{2} k[y + 2x] \]

16. (a) Given, radius of first soap bubble \( (R_1) = 2 \text{ cm} \)
Radius of second soap bubble \( (R_2) = 1 \text{ cm} \)

We know that,
Radius of another bubble having same pressure difference,
\[ R = \frac{R_1 R_2}{R_1 + R_2} \text{ or } R = \frac{2.0 \times 10^{-4}}{(2.0 + 1.0) \times 10^{-2}} \]
or
\[ R = \frac{2 \times 10^{-4}}{3 \times 10^{-2}} \text{ or } R = 6.67 \times 10^{-3} \text{ m} \]

17. (d) Given, area of slab of stone \( (A) = 3600 \text{ cm}^2 \)
\[ = 3600 \times 10^{-4} \text{ m}^2 \]

Thickness of stone slab = 10 cm = \( 10 \times 10^{-2} \) m
Temperature difference \( (\Delta T) = 100^\circ C \)
As, heat radiated from black body surface,
\[ \frac{Q}{t} = \frac{KA\Delta T}{l} \]
\[ \Rightarrow \frac{48 \times 336 \times 10^5}{60 \times 60} = K \times 3600 \times 10^{-4} \times 100 \]
18. (a) Given,
Temperature of black body \( T \) = 727°C
\( T = 727 + 273 = 1000 \) K
Cross-sectional area of black body \( A \) = 1 m²
Stefan’s constant \( \sigma \) = \( 5.7 \times 10^{-8} \) W/m²K⁴

We know that
Heat radiated per second
\[ E = \sigma A T^4 = 5.7 \times 10^{-8} \times 1 \times (1000)^4 \]
or
\[ E = 57000 \text{ J} \]
∴ Heat radiated from this surface in one minute,
\[ E = 57000 \times 60 \]
or
\[ E = 3.42 \times 10^6 \text{ or } 34.2 \times 10^5 \text{ J} \]

19. (d) Given,
In the first condition, work done \( W_1 \)
In the second condition, work done \( W_2 \)

Now, according to the question,
In the first condition,
\[ W_1 = p(2V - V) \]
or
\[ W_1 = pV \] \hspace{1cm} (i)

In the second condition,
\[ W_2 = nR T \log_{\frac{V_2}{V_1}} \]
or
\[ W_2 = pV \log_{\frac{2V}{V}} \] \hspace{1cm} [∵ \( pV = nRT \)]

From Eq. (i),
\[ W_2 = W_1 \log_u(2) \]

20. (c) Given,
Mass of first uranium isotope = 238 units
Mass of second uranium isotope = 235 units

We know that,
\[ v_{rms} = \sqrt{\frac{nRT}{M}} \Rightarrow v_{rms} \propto \frac{1}{\sqrt{M}} \]

So, the percentage ratio of difference in rms velocity of isotopes is given by
\[ \frac{v_{rms_2} - v_{rms_1}}{v_{rms_1}} \times 100 \]
\[ = \frac{\sqrt{M_1} - \sqrt{M_2}}{\sqrt{M_1}} \times 100 \]
\[ = \frac{\sqrt{238} - \sqrt{235}}{\sqrt{238}} \times 100 \]
\[ = \frac{15.427 - 15.329}{15.427} \times 100 \approx 0.634 \times 100 = 0.634 \times 10^1 \]

21. (d) The frequency of source, \( f_s = 340 \text{ Hz} \)
Velocity of sound = 340 m/s
Length of the tube = 120 cm
The minimum height of the tube for resonance, \( l = \frac{v}{4f} = \frac{340}{4 \times 340} \text{ m} \)
\[ = \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm} \]

From second resonance condition for closed pipe,
\[ h = l_2 - 3l \]
Here,
\[ l_2 = 120 \text{ cm} \]
∴
\[ h = 120 - 3 \times 25 = 120 - 75 \]
\[ = 45 \text{ cm} = 0.45 \text{ m} \]
So, for second resonance, 0.45 m should be the height of water level. It will be the minimum height of water in the tube.

22. (c) When the convex lens of focal length \( f \) made of crown glass immersed in liquid of refractive index \( \mu \), \( \mu_u \) is the refractive index of crown glass, thus from lens maker’s formula, we get focal length of the lens as negative \( \mu \) also. Therefore, the convex lens will behave like a concave lens. So, it will be a divergent lens.

23. (b) The linear magnification of lens in terms of focal length \( f \) and \( u \) is given as
\[ m = \frac{f}{f + u} \]

For first lens,
\[ u_1 = \left( \frac{1 - \frac{m_1}{m}}{m_1} \right) \]

For second lens,
\[ u_2 = \left( \frac{1 - \frac{m_2}{m}}{m_2} \right) \]

According to the question,
\[ \frac{m_1}{m} = \frac{m_2}{m} \]
∴
\[ \frac{u_1}{u_2} = \frac{m_1 - 1}{m_2 - 1} \]

24. (b) Key Idea The limit of resolution of a telescope is
\[ \theta = \frac{1.22 \lambda}{d} \]
where, \( \lambda \) = wavelength of light rays
\( d \) = aperture of objective lens of telescope.
Here, \( \lambda = 6400 \text{ Å} = 6400 \times 10^{-10} \text{ m} \)
\( d = 200 \text{ cm} = 2 \text{ m} \)

Substitution of values gives
As, limit of resolution of a telescope,
\[ \theta = \frac{1.22 \times 6400 \times 10^{-10}}{2} = \frac{1.22 \times 64 \times 10^{-8}}{2} \]
\[ = 1.22 \times 32 \times 10^{-8} = 39 \times 10^{-8} \text{ rad} \]
25. (b) Spheres A and B have the same charge of same nature. When uncharged sphere C is touched with sphere A, then charge is transferred to C and this charge is equally divided between two spheres.

\[ F_{BC} = \frac{kq \times q}{(r/2)^2} \]

Similarly, between B and C
\[ F_{BC} = \frac{kq \times q}{(r/2)^2} \]

The magnitude of net force on C, \( F' = F_{BC} - F_{AC} \)

Also \( F = \frac{kq \times q}{r^2} = 3 \times 10^{-5} \text{ N} \)

So, \( F_{BC} - F_{AC} = \frac{kq \times q}{(r/2)^2} - \frac{kq \times q}{(r/2)^2} = \frac{kq \times q}{(r/2)^2} \)

Here, \( F = 3 \times 10^{-5} \text{ N}, F_{BC} - F_{AC} = F' \)

\[ 3 \times 10^{-5} = \frac{1}{2} - \frac{1}{1} = 1 \]

\( F' = 3 \times 10^{-5} \text{ N} \)

26. (b) Let the volume charge density be \( \rho \).

The electric field inside the charged sphere,

\[ E = \frac{K q r}{R^3} \]

where, \( R = \text{Radius of charged sphere} \)

\( r = \text{Distance of the point (P) inside the sphere} \)

As volume charge density is \( \rho \), then in terms of charge density,
\[ E = \frac{\rho r}{3\varepsilon_0} \]

Now, electric field, \( E = -\frac{dV}{dr} \)

Here, \( V = Ar^2 + B \)

\[ E = -\frac{dV}{dr} = -2Ar + 0 = -2Ar \]

\[ \rho = \frac{3\varepsilon_0}{r} \]

27. (d) Key Idea In parallel combination, the equivalent resistance is given by

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

where, \( R_1, R_2, R_3 \) are resistances and \( R_{eq} \) is equivalent resistance.

According to the question, equivalent resistance in parallel

\[ = 1 \Omega \]

Here,

\[ R_{eq} = 1 \]

\Rightarrow \frac{1}{1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

Let \( R_3 = 2R_2 \)

If we check the option given in the question
\[ \frac{1}{1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

\[ \frac{1}{1} = \frac{1}{2R_2} + \frac{1}{R_2} + \frac{1}{2R_2} \]

\[ \frac{1}{1} = \frac{1 + 3}{2R_2} \]

\[ \frac{1}{1} = \frac{1 + 3}{4} = \frac{7}{4} \]

will not hold the condition.

Similarly, if we check the other options only
\[ \frac{1}{R_3} = \frac{1}{2} + \frac{1}{3} + \frac{1}{R_3} \]

\[ \frac{5}{6} + \frac{1}{R_3} \Rightarrow 1 - \frac{5}{6} = \frac{1}{R_3} \]

or \( \frac{1}{6} = \frac{1}{R_3} \) or, \( R_3 = 6\Omega \)

So, it will satisfy the equation,
\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

or \( \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{1}{1} \)

or \( R_{eq} = 1\Omega \)

\( R_3 = 6\Omega \) will be highest resistance.

28. (d) Power generated in resistor \( P = \frac{V^2}{R} \)

\[ R = \frac{V^2}{P} \]

Here,
\[ P = 320 \, \text{W} \]

\[ V = 220 \, \text{V} \]

When the resistors are connected in series, then power developed across each resistor,
\[ P' = \left(\frac{V'}{R}\right)^2 = \frac{110^2}{2} \times \frac{R}{R} = \frac{110 \times 110}{2 \times 2} \times \frac{P}{V^2} \]

\[ = \frac{110 \times 110}{2} \times \frac{320}{220 \times 220} = \frac{1}{4} \times 320 = 80 \, \text{W} \]
29. (b) Let the centroid of the triangle is O, magnetic field due to side AB

\[ B_i = \frac{\mu_0 i}{4\pi} \frac{1}{r} (\sin \phi_1 + \sin \phi_2) \]

where, \( r \) is perpendicular distance of AB from centroid O. The magnetic field due to each side is same in magnitude and direction, so the resultant field

\[ B = 3B_i = \frac{3\mu_0 i}{4\pi} \frac{1}{r} (\sin \phi_1 + \sin \phi_2) \]

[Here, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)]

\[ i = 1 \text{ A} \]

In equilateral triangle, \( \phi_1 = \phi_2 = 60^\circ \)

\[ B = \frac{3\mu_0 i}{4\pi} \frac{1}{r} (\sin 60^\circ + \sin 60^\circ) \]

Also, \( r = 4.5 \times 10^{-2} \text{ m} \)

\[ \Rightarrow B = \frac{3\mu_0 i}{4\pi} \frac{1}{4.5 \times 10^{-2}} \left( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) \text{T} \]

\[ = \frac{3\sqrt{3}}{4\pi} \mu_0 \frac{1}{4.5 \times 10^{-2}} \text{T} \]

\[ = \frac{3\sqrt{3} \times 10^{-7}}{4.5 \times 10^{-2}} \times 10^{-5} \text{T} \]

\[ = 1.15 \times 10^{-5} \text{T} \]

\[ T = 2 \times 10^{-5} \text{T} \]

30. (a) The magnetic moment of charge, \( \mu = niA \)

31. (a) **Key Idea**

The time period of oscillation of the magnet is given as

\[ T = 2\pi \sqrt{\frac{l}{MBH}} \]

where, \( l = MI \) of magnet about the axis passing through its centre

\( BH = \) Earth’s magnetic field

\( M = \) Magnetic moment of magnet

Let the mass of the first magnet be \( m \).

\[ \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2} M_1} \]

Let length of the magnet be \( L \).

MI of the magnets about the axis passing through one of the ends is \( \frac{ml^2}{3} \)

\[ \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{ml^2}{3}} \times \frac{M_2}{2m \times (2L)^2} \]

It is given that maximum torques experienced by the magnets are same.

\[ \Rightarrow \frac{M_1 BH}{M_2 BH} \]

\[ \Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \]

32. (b) Mutual inductance, \( M = 0.005H \)

\[ l = l_0 \sin \omega t \]

\[ l_0 = 10 \text{ A} \]

\[ \omega = 100\pi \text{ rad/s} \]

The emf induced in second coil,

\[ e = M \frac{dl}{dt} \]

\[ e = M \frac{dl}{dt} = 0.005 \times l_0 \sin \omega t \]

\[ = 0.005 \times 10 \text{ A} \cos \omega t \]

\[ = 0.005 \times l_0 \cos \frac{2\pi}{l} t \]

\[ \Rightarrow \text{Maximum value of induced emf} \]

\[ e_{\text{max}} = 0.005 \times 10 \times 100 \times \pi \times 1 \]

\[ = 5\pi \text{ volt} \]
33. (c) Capacitance, \(C = \frac{10^{-3}}{2\pi f}\) F

Inductance, \(L = \frac{100}{\pi}\) mH = \(\frac{100}{\pi} \times 10^{-3}\) H

Resistance, \(R = 100\) Ω

Voltage of source = 220 V

Frequency of source, \(f = 50\) Hz

Phase angle \(\phi\) is given by \(\tan \phi = \frac{X_L - X_C}{R}\)

where, \(X_L = \text{inductive impedance} = \omega L\)
\[\therefore X_L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10 \Omega\]

and \(X_C = \text{capacitive impedance}\)
\[= \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1 \times 2\pi}{2\pi \times 50 \times 10^{-3}} = \frac{100}{5} = 20 \Omega\]

\[\therefore \text{Phase angle of the circuit,} \quad \tan \phi = \frac{X_C - X_L}{R} = \frac{20 - 10}{10} = 1\]

or \(\phi = 45^\circ\)

34. (b) \(\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}\)

It is Gauss law of electrostatic fields.

(B) \(\oint \vec{B} \cdot d\vec{A} = 0\)

Because single pole of a magnet does not exist. So, it is Gauss law for magnetic fields. Option (b) is correct.

35. (b) Key Idea The de-Broglie wavelength associated with a charged particle is given as
\[\lambda = \frac{\hbar}{mqV}\]

where, \(\hbar = \text{Planck’s constant}\)
\(m = \text{mass of charged particle}\)
\(q = \text{charge on the particle}\)
\(V = \text{potential at which the charged particle is accelerated}\).

\[\therefore \lambda_1 = \frac{\hbar}{\sqrt{m_1qV_1}}, \quad \lambda_2 = \frac{\hbar}{\sqrt{m_2qV_2}}\]

Here, \(m_1 = m_2\)
\[\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{m_1qV_1}}{\sqrt{m_2qV_2}} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{V_2}{V_1}}\]

36. (c) Key Idea From hydrogen spectrum, when electron transit from \(n_2\) orbit to \(n_1\) orbit, the emitted wavelength is given by the formula
\[\lambda_L = \frac{R Z^2}{(n_1^2 - n_2^2)}\]

where, \(R\) is Rydberg constant.

For Lyman series, \(n_1 = 1, n_2 = 2\)

For first Lyman line, \(\frac{1}{\lambda_L} = R Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)\)
\[\therefore \frac{1}{\lambda_L} = \frac{3}{4} R Z^2\] \(\ldots \) (i)

For Balmer series, \(n_1 = 2, n_2 = 3, 4, \ldots \)

For first Balmer line,
\[\frac{1}{\lambda_B} = R Z^2 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)\]
\[= \frac{R Z^2}{27}\]
\[\frac{1}{\lambda_B} = \frac{R Z^2}{36}\]
\[\therefore \lambda_B = \frac{27}{5}\]
\[\lambda_B = \frac{27}{5} \times 1215.4 \AA = 6563.16 \AA = 6563 \AA\]

37. (a) Decay constant, \(\lambda = 7.9 \times 10^{-10} \text{ / second}\)

Activity, \(A = 55.3 \times 10^{11} \text{ disintegration / second}\)

Let the number of nuclei at that instant be \(N\).

We know activity, \(A = \lambda N\)
\[\therefore N = \frac{A}{\lambda} = \frac{55.3 \times 10^{11}}{7.9 \times 10^{-10}} = 7.0 \times 10^{21}\]

38. (a) Change in current through junction diode, \(\Delta I = 1.2 \text{ mA} = 1.2 \times 10^{-3} \text{ A}\)

Change in forward bias voltage, \(\Delta V = 0.6 \text{ V}\)

Dynamic resistance,
\[R_{\text{dyn}} = \frac{\Delta V}{\Delta I} = \frac{0.6}{1.2 \times 10^{-3}} = 500 \Omega\]

39. (a) Electron concentration, \(n_e = 2 \times 10^8 \text{ m}^{-3}\)

Hole concentration, \(n_h = 2 \times 10^8 \text{ m}^{-3}\)

After doping with an impurity, the new electron concentration, \(n_e = 4 \times 10^{10} \text{ m}^{-3}\)

New hole concentration, \(n_h’\)

We know that, \(n_e \times n_h = n_e^2\)

and \(n_e’ \times n_h’ = n_e^2\)
\[\therefore n_h’ = \frac{n_e^2}{n_e’} = \frac{n_e \times n_h}{n_h’}\]
\[n_h’ \times n_h = n_e \times n_h’\]
\[= 2 \times 10^8 \times 2 \times 10^8 / 4 \times 10^{16} = 10^8 / m^3\]

So, concentration of holes in semiconductor,
\[n_h’ = 10^8 \text{ m}^{-3}\]

40. (c) Given, message signal’s peak voltage = 20 V
\[E_m = 20 \text{ V}\]

Carrier voltage, \(E_c = 30 \text{ V}\)

Thus, modulation index,
\[m_a = \frac{E_m}{E_c} = \frac{20}{30} = 0.67\]
**GI-FI TECHNOLOGY**

**GI-FI Technology**
GI-FI is abbreviated as Gigabit Fidelity or Gigabit Wireless. It consists of a chip which has 60 GHz configuration and the facility to deliver short range multi-gigabit data transfer in a local environment and ten times faster as compared to other technologies such as Wi-Fi, bluetooth, etc.

- **High speed of data transfer**
- **High security**
- **Small size**
- **Low power consumption**
- **Cost effective**
- **Highly portable, high mobility**

**Comparison of Gi-Fi and Existing Technologies**

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<th>Wi-Fi</th>
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<td>IEEE (Institute of Electrical and Electronics Engineers)</td>
<td>NICTA (National Information and Communications Technology Australia Ltd.)</td>
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<td>Note book computers, Desktop computers, servers.</td>
<td>Mobile phones, home devices, PDAs, consumer, electronics, offices, industrial automation devices</td>
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<td>3. Power consumption</td>
<td>5 MW</td>
<td>10 MW</td>
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<td>4. Data transfer rate</td>
<td>800 kbps</td>
<td>11 Mbps</td>
<td>5 Gbps</td>
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<tr>
<td>5. Range</td>
<td>10 m</td>
<td>100 m</td>
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<td>6. Frequency</td>
<td>2.4 GHz</td>
<td>2.4 GHz</td>
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<td>7. Speed</td>
<td>Low</td>
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</table>
1. Capacitor

A capacitor is an arrangement for storing large amounts of electric charge and hence, electric energy in a small space. Usually, it consists of a system of two conductors separated by an insulating medium.

The property of a capacitor that characterises its ability to store energy is called its capacitance.

2. Various Type of Capacitors

(a) Parallel plate capacitor

A parallel plate capacitor consists of two large plates placed parallel to each other with a separation d smaller in comparison to the length and breadth of plates.

Potential difference \( V = \frac{Q}{C} \)

Potential difference between the plates is \( V = Ed = \frac{Q}{A} \) and capacitance is \( C = \frac{Q}{V} = \frac{Q}{Ed} \)

(b) Spherical capacitor

Two concentric spherical conducting shells of radii a and b i.e. \( b > a \). Potential difference, \( V_b - V_a = \frac{\rho}{\epsilon_0} \) E dr = \( \frac{Q}{4\pi\epsilon_0} \) r dr. When \( V_b = 0 \), then capacitance \( C = \frac{Q}{V_a} = \frac{Q}{V_b} = \frac{4\pi\epsilon_0ab}{b-a} \)

(c) Cylindrical capacitor

Two coaxial cylinders of radii a and b i.e. \( b > a \). The outer radius b is earthed.

Potential difference, \( V_b - V_a = \frac{\rho}{\epsilon_0} \) E dr = \( \frac{Q}{2\pi\epsilon_0} \) ln(b/a)

When \( V_b = 0 \), then \( V_a = \frac{Q}{2\pi\epsilon_0} \ln(b/a) \)

and capacitance is \( C = \frac{Q}{V_a} = \frac{Q}{V_b} = \frac{2\pi\epsilon_0ab}{\ln(b/a)} \)

3. Grouping of Capacitors

Capacitors in series

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]

- Total potential drop V across the combination of capacitors, \( V = V_1 + V_2 + V_3 \)
- Charge on all capacitors is same.
- Resultant capacitance, \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \)

Capacitors in parallel

\[ Q_1 + Q_2 + Q_3 \]

- Total charge Q across the capacitors.
- Charge on all capacitors is same.
- Resultant capacitance, \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \)

4. Energy stored in a capacitor

A capacitor is a charged conductor with a stored charge at a lower potential.

\[ W = \frac{1}{2} CV^2 \]

- Small amount of w additional charge dQ is stored in the capacitor.
- Work done in giving dQ = \( Q \times dq \) to the condenser \( W = Q/dq \)

5. Dielectric Medium

Dielectric are non-conducting substances when it is held in an electric field, small induced charges appear on its surface. However, there is no free movement of charges inside a dielectric.

Polar dielectrics

- Polar dielectrics, e.g., H₂O, HCl are made of polar molecules in which centre of positive charges does not coincide with centre of negative charge and each molecule has some intrinsic electric dipole moment.

Non-polar dielectrics

- Non-polar dielectrics, e.g., O₂, N₂, CO₂ consist of non-polar molecules in which centre of positive charge exactly coincides with centre of negative charge and dipole moment of a molecule is zero.

6. Effect of Dielectric

Induced charge density \( \sigma = \epsilon_0 \chi \)

Susceptibility of dielectric \( \chi = \epsilon_0 (K - 1) \)

7. Capacitance of a parallel capacitor with a conductor

When a conducting slab thickness \( t = d \) is introduced between the plates, then charges \( Q \) and \( -Q \) appear on the two faces of the slab as well as the space in the field in the slab.

\[ V = \frac{Q}{C} \]

- Field with dielectric \( E = \frac{\sigma}{\epsilon_0} \)
- Field with air between plates \( E = \frac{\sigma}{\epsilon_0} \)
- Potential difference between plates \( V_0 = \frac{\sigma}{\epsilon_0}d \)
- Capacity, \( C_0 = \frac{\sigma}{\epsilon_0}d \)

8. Conducting slab

So, potential difference \( V = \frac{Q}{C_0} \)

- i.e., capacitance increases if a conducting slab is introduced.
**VISUALISE PHYSICS**

**ENHANCE YOUR PHYSICS LEARNING THROUGH FIGURES**

**CAPACITOR AND CAPACITANCE**

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7. **Insertion of Dielectric Slab in a Charged Capacitor**

- **When charge is held constant**
  - i.e., Battery is disconnected
  - Capacitor is charged and battery is removed
  - Induced charges appear on dielectric and it is pulled in by the field

- **When potential is held constant**
  - i.e., Battery remains connected
  - Battery remains connected
  - As dielectric is pulled in, battery kept same

---

9. **Capacitance of a parallel plate capacitor with a dielectric slab**

- When a dielectric slab of thickness \( t \) is introduced between the plates, then molecules in the slab get polarised in the direction of \( E_0 \). Therefore, effective field inside the dielectric is \( E = E_0 - E_D \).

- Potential difference between the two plates is \( V = E_0(d-t) + E_D(t+d) \).

- Capacitance of the capacitor with dielectric in between the plate is \( C = \frac{Q}{V} = k_0 \varepsilon_0 A/d + [1 - 1/k] \).

---

10. **Kirchhoff's rules for capacitors**

**Junction rule**

- In any isolated system, the net charge is conserved, i.e., \( q = q_1 + q_2 \).

**Loop rule**

- In closed loop, algebraic sum of the potential difference is equal to zero, i.e., \( \sum V = 0 \).
SPECTRUM

PERSONALITIES

MICHAEL FARADAY

“The lecturer should give the audience full reason to believe that all his powers have been exerted for their pleasure and instruction.”

LIFE HISTORY

Michael Faraday (1791-1867) was an English scientist who contributed to the study of electromagnetism and electrochemistry. His main discoveries include the principles underlying induction, diamagnetism and electrolysis. Although Faraday received little formal education, he was one of the most influential scientist in history and also an excellent experimentalist who conveyed his ideas in clear and simple language of the Royal Institution and the Royal Society and John Tatum, founder of the City Philosophical Society. During his lifetime, her was offered knighthood in recognition for his services to science, which he turned down on religious grounds. He died at his house at Hampton Court in 1867 at the age of 75.

CONTRIBUTIONS

• Faraday proved the basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an Electromotive Force (EMF).
• He determined the Faraday effect, the first experimental evidence that proved light and electromagnetism are related.
• The Faraday constant, denoted by the symbol F and named after Michael Faraday is the magnitude of electric charge per mole of electrons that has value of 96485.33289(59) C mol⁻¹.
• Faraday waves, non-linear standing waves firstly described them in an appendix to an article in the Philosophical Transactions of the Royal Society of London.
• Invented the Faraday rotator that works on the principle of a magneto-optic effect.

AWARDS AND HONOURS

• In 1835 and 1846, Royal Medal.
• In 1832 and 1838, Copley Medal.
• In 1833, the University of Oxford granted Faraday a Doctor of Civil Law degree (honorary).
• In 1833, Faraday became the first Fullerian Professor of Chemistry at the Royal Institution of Great Britain.
• In 1846, Rumford Medal.
• In 1866, Albert Medal.
• In 1986, Michael Faraday Prize, named in his honour awarded annually to the scientist or engineer whose expertise in communicating scientific ideas at the Royal Institution.

WILLIAM THOMSON

“There is nothing new to be discovered in physics now, all that remains is more and more precise measurement.”

LIFE HISTORY

William Thomson, (1824-1907) was an Irish mathematician and engineer who was born in Belfast in 1824. At the University of Glasgow he did important work in the mathematical analysis of electricity and formulation of the first and second laws of thermodynamics and did much to unify the emerging discipline of physics in its modern form. He worked closely with mathematics professor Hugh Blackburn in his work. He also had a career as an electric telegraph engineer and inventor, which propelled him into the public eye and ensured his wealth, fame and honour. For his work on the transatlantic telegraph project he was knighted by Queen Victoria, becoming Sir William Thomson. He had extensive maritime interests and was most noted for his work on the mariner’s compass, which had previously been limited in reliability. Always active in industrial research and development, he was recruited around 1899 by George Eastman to serve as vice-chairman of the board of the British company Kodak Limited, affiliated with Eastman Kodak.

CONTRIBUTIONS

• Joule-Thomson effect that describes the temperature change of a real gas or liquid as differentiated from an ideal gas.
• Invented the Kelvin double bridge to measure unknown electrical resistors below 1 Ω.
• Thomson invented the absolute temperature scale, now known as ‘the Kelvin scale’ that would be beneficial to define extremely low temperatures precisely.
• His experimental research, works on the formulation of second law of thermodynamics which states that heat will not flow from a colder body to a hotter body and first formulated to explain how a steam engine works.
• Kelvin described the theory about the shape of atoms in which he observed smoke rings, and proposed that atoms were shaped like vortices spiralling around each other similar to the way knots loop and twist.
• Analysis of the correspondence principles and basic tools related to the Kelvin-Helmholtz mechanism and Kelvin transform.

AWARDS AND HONOURS

• 1845, First Smith’s Prize.
• 1851, Foreign member of the Royal Swedish Academy of Sciences.
• 1856, Royal Medal.
• 1859, Hon. Member of the Institution of Engineers and Shipbuilders in Scotland.
• 1864, Keith Medal.
• 1873, Commander of the Imperial Order of the Rose (Brazil).
• 1896, Knight Grand Cross of the Victorian Order.
• 1901, Order of the First Class of the Sacred Treasure of Japan.
• 1905, First international recipient of John Fritz Medal.
COMPREHENSION TYPE QUESTIONS ON KINEMATICS

A collection of chapterwise best problems of their types

Passage 1

The graph below gives the displacement of a particle travelling along the $X$-axis as a function of time. $AM$ is the tangent to the curve at the starting moment and $BN$ is tangent at the end moment ($\theta_1 = \theta_2 = 120^\circ$).

1. Find the average velocity during the first 20 s.
   (a) $-10$ m/s  
   (b) $10$ m/s  
   (c) Zero  
   (d) None of these

2. Find the average acceleration during the first 20 s.
   (a) $\sqrt{3}$ m/s$^2$  
   (b) $-\sqrt{3}$ m/s$^2$  
   (c) Zero  
   (d) $1$ m/s$^2$

3. During which interval is the motion retarded.
   (a) 0-10  
   (b) 5-15  
   (c) 10-20  
   (d) None of these

Passage 2

The direction of velocity of a particle at time $t = 0$ is as shown in the figure and has the magnitude $u = 20$ m/s. The acceleration of particle is always constant and has magnitude $10$ m/s$^2$. The angle between its initial velocity and acceleration is $127^\circ$. (Take, $\sin 37^\circ = 3/5$).

4. The instant of time at which acceleration of particle is perpendicular to its displacement (displacement from $t = 0$ till that instant) is
   (a) $0.6$ s  
   (b) $1.2$ s  
   (c) $2.4$ s  
   (d) None of these

Passage 3

When two bodies $A$ and $B$ are moving with velocity $v_A$ and $v_B$, then relative velocity of $A$ w.r.t. $B$ is $v_{AB} = v_A - v_B$.

Relative velocity of $B$ w.r.t. $A$ is $v_{BA} = v_B - v_A = v_B + (-v_A)$.

When body $C$ is moving with velocity $v_c$ on a body $A$, which is moving with velocity $v_A$, then velocity of $C$ w.r.t. ground is $v_c + v_A$.

Suppose two parallel rail tracks run north-south. Train $A$ moves north with a speed of $54$ km/h and train $B$ moves south with a speed of $90$ km/h.

5. Relative velocity of ground w.r.t. $B$ is
   (a) $25$ ms$^{-1}$ due north  
   (b) $25$ ms$^{-1}$ due south  
   (c) $40$ ms$^{-1}$ due north  
   (d) $40$ ms$^{-1}$ due south

6. The instant of time at which acceleration and velocity are perpendicular is
   (a) $0.6$ s  
   (b) $1.2$ s  
   (c) $2.4$ s  
   (d) None of these

7. The instant of time at which speed of particle is least
   (a) $0.6$ s  
   (b) $1.2$ s  
   (c) $2.4$ s  
   (d) None of these

8. A monkey is moving with a velocity of $18$ km/h on the roof of train $A$ against the motion of train $A$. The velocity of monkey as observed by a man standing on the ground is
   (a) $5$ ms$^{-1}$ towards south  
   (b) $10$ ms$^{-1}$ towards north  
   (c) $10$ ms$^{-1}$ towards south  
   (d) $20$ ms$^{-1}$ towards south

Passage 4

A man wants to cross a river of width $d$. He wants to reach at the opposite point $B$. If $v_m = 10$ m/s and $v_r = 12$ m/s, then solve the question given below.

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9. The value of angle made by the velocity of man by line 
    \( AB \).
    \( (a) \ \theta = \sin^{-1}\left(\frac{10}{12}\right) \quad (b) \ \theta = \sin^{-1}\left(\frac{12}{10}\right) \quad (c) \ 60° \quad (d) \ None \ of \ these \)

10. If \( v_m \) becomes 24 m/s, then the time taken to reach the
    exactly opposite point \( B \).
    \( (a) \ \frac{5}{6}\sqrt{3} \text{ m/s} \quad (b) \ \frac{5}{6} \text{ m/s} \quad (c) \ 3\sqrt{3} \text{ m/s} \quad (d) \ None \ of \ these \)

**Passage 5**

A particle \( A \) is projected with an initial velocity of 60 m/s
    at an angle of 30° to the horizontal. At the same time, a
    second particle \( B \) is projected in opposite direction with
    initial speed of 50 m/s from a point at a distance of 100 m
    from \( A \). If the particles collide in air, then

**Diagram:**

- Particle \( A \) projected with 60 m/s at 30°.
- Particle \( B \) projected with 50 m/s in opposite direction.

11. Find the angle of projection \( \alpha \) of particle \( B \).
    \( (a) \ \alpha = \sin^{-1}\left(\frac{4}{5}\right) \quad (b) \ \alpha = \sin^{-1}\left(\frac{3}{5}\right) \quad (c) \ \cos^{-1}\left(\frac{4}{5}\right) \quad (d) \ \cos^{-1}\left(\frac{3}{5}\right) \)

12. Find the time period when the collision takes place.
    \( (a) \ t = 2.0 \text{ s} \quad (b) \ t = 1.5 \text{ s} \quad (c) \ t = 1.09 \text{ s} \quad (d) \ t = 1.45 \text{ s} \)

13. Find the distance of \( P \) from \( A \), where collision occurs.
    \( g = 10 \text{ m/s}^2 \)
    \( (a) \ s = 49.52 \text{ m} \quad (b) \ s = 62.64 \text{ m} \quad (c) \ s = 60.42 \text{ m} \quad (d) \ s = 64 \text{ m} \)

**Passage 6**

Two inclined planes \( OA \) and \( OB \) having inclinations 30°
    and 60° with the horizontal respectively intersect each
    other at \( O \) as shown in figure. A particle is projected from
    point with velocity \( u = 10\sqrt{3} \) m/s along a direction
    perpendicular to plane \( OA \). If the particle strikes the
    plane \( OB \) perpendicular at \( Q \). Calculate

**Diagram:**

- Plane \( OA \) at 30°.
- Plane \( OB \) at 60°.

14. Find the time period of flight.
    \( (a) \ t = 2 \text{ s} \quad (b) \ t = 6 \text{ s} \quad (c) \ t = 4 \text{ s} \quad (d) \ t = 5 \text{ s} \)

15. Find the velocity with which the particle strikes the
    plane \( OB \).
    \( (a) \ v = 10 \text{ m/s} \quad (b) \ v = 5 \text{ m/s} \quad (c) \ v = -10 \text{ m/s} \quad (d) \ v = -5 \text{ m/s} \)

16. Find the height \( h \) of point \( P \) from point \( O \).
    \( (a) \ h = 5 \text{ m} \quad (b) \ h = 8 \text{ m} \quad (c) \ h = 10 \text{ m} \quad (d) \ h = 4 \text{ m} \)

17. Find the distance \( PQ \). (\( g = 10 \text{ m/s}^2 \))
    \( (a) \ 20 \text{ m} \quad (b) \ 15 \text{ m} \quad (c) \ 10 \text{ m} \quad (d) \ 5 \text{ m} \)

**Passage 7**

An elevator car whose floor to ceiling distance is equal to
    2.7 m starts ascending with constant acceleration 1.2 m/s^2.
    2 second after the start a bolt begins falling from the ceiling
    of the car.

18. Find the time after which bolts hits the floor of the
    elevation.
    \( (a) \ 0.8 \text{ s} \quad (b) \ 0.7 \text{ s} \quad (c) \ 0.5 \text{ s} \quad (d) \ 0.4 \text{ s} \)

19. Find the net displacement and distance travelled by the
    bolt with respect to earth.
    \( (a) \ (-0.72 \text{ m}, 1.3 \text{ m}) \quad (b) \ (-0.72 \text{ m}, 1.5 \text{ m}) \quad (c) \ (-0.48 \text{ m}, 1.3 \text{ m}) \quad (d) \ (-0.52 \text{ m}, 1.4 \text{ m}) \)

**Passage 8**

Two guns, situated on the top of a hill of height 10 m, fire on
    one shot each with the same speed \( 5\sqrt{3} \text{ m/s} \) at some
    interval of time. One gun fires horizontally and other fires
    upwards at an angle of 60° with the horizontal. The shots
    collide in air at a point \( P \).

20. Find the time interval between the firings.
    \( (a) \ 1 \text{ s} \quad (b) \ 4 \text{ s} \quad (c) \ 2 \text{ s} \quad (d) \ 3 \text{ s} \)

21. Find the coordinates of the point \( P \).
    \( (a) \ (5\sqrt{3}, 4) \text{ m} \quad (b) \ (5\sqrt{3}, 5) \text{ m} \quad (c) \ (4\sqrt{3}, 2) \text{ m} \quad (d) \ (6 \sqrt{3}, 2) \text{ m} \)

**Passage 9**

A rider on an open platform, which is descending at a
    constant speed of 3 ms^{-1}, throws a ball. Relative to the
    platform, ball’s initial velocity is horizontal at 12 m/s. The
    ground is 10 m below the location where the ball is thrown.

22. Find the distance where ball hits the ground.
    \( (a) \ 13.80 \text{ m} \quad (b) \ 14.24 \text{ m} \quad (c) \ 12.45 \text{ m} \quad (d) \ 12.42 \text{ m} \)

23. How long after the ball hits the ground does the platform
    reach the ground level?
    \( (a) \ t = 2.45 \text{ s} \quad (b) \ t = 1.45 \text{ s} \quad (c) \ t = 2.18 \text{ s} \quad (d) \ t = 2.170 \text{ s} \)
24. With what velocity does the ball hits the ground?
   (a) \( v = 18.68 \text{ m/s} \)  
   (b) \( v = 15.40 \text{ m/s} \)  
   (c) \( v = 20.00 \text{ m/s} \)  
   (d) \( v = 18.00 \text{ m/s} \)

Passage 10
A bullet of mass \( M \) is fired with a velocity 50 m/s at an angle \( \theta \) with the horizontal. At the highest point of its trajectory, it collides head on with a bob of mass 3\( M \) suspended by a massless string of length \( (10/3) \text{ m} \) and gets embedded in the bob. After the collision, the string moves through an angle of 120°.

25. Find the value of angle \( \theta \).
   (a) \( \theta = 30^\circ \)  
   (b) \( \theta = 45^\circ \)  
   (c) \( \theta = 60^\circ \)  
   (d) \( \theta = 35^\circ \)

26. Find the vertical and horizontal coordinates of the initial position bob w.r.t. to the point of firing of the bullet. \( (g = 10 \text{ m/s}^2) \)
   (a) (108.25 m, 31.25 m)  
   (b) (108.14 m, 30.0 m)  
   (c) (105.25 m, 30.45 m)  
   (d) (100 m, 35.27 m)

Answers with Explanation

1. (a) Average velocity, \( \mathbf{v} = \frac{x_f - x_i}{\Delta t} = \frac{-100 - 100}{20} = -10 \text{ m/s} \)

2. (c) Average acceleration, \( \mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \frac{\tan \theta - \tan \theta}{20} = 0 \) \( (\because \theta = \theta) \)

3. (a) During first 10 s, magnitude of the slope of \( x-t \) curve and hence, speed is decreasing motion is retarded.

4. (c) Acceleration and displacement are mutually perpendicular at instant \( t_0 = 2.4 \text{ s} \).

5. (a) Relative velocity of ground w.r.t. \( B \)
   \[ \mathbf{v}_{GB} = \mathbf{v}_B + (-\mathbf{v}_G) = 0 + (-25 \text{ m/s}) \]
   \[ = 25 \text{ m/s} \text{ due north.} \]

6. (b) As, the path of particle is parabolic as shown in figure.

\[ \mathbf{u} = 20 \text{ m/s} \]
\[ \theta = 37^\circ \]
\[ a = 10 \text{ m/s}^2 \]
\[ \therefore \mathbf{a} \perp \mathbf{v} \text{ at maximum height, that is at half time of flight.} \]
Hence, \( t_0 = \frac{u \sin \theta}{a} = \frac{20 \times 3}{5} = 1.2 \text{ s} \)

7. (b) Speed is least at maximum height, that is at instant \( t_0 = 1.2 \text{ s} \).

8. (b) Velocity of monkey as observed by a man,
   \[ v_{mG} = v_m + v_G = 5 + (-15 \text{ m/s}) \]
   \[ = -10 \text{ m/s} \text{ towards south} \]
   \[ = 10 \text{ m/s} \text{ towards north} \]

9. (b) To reach at \( B \), the displacement along \( X \)-axis should be zero.
   For that \( v_m \sin \theta \) must be equal to \( v_r \).
   \[ v_m \sin \theta = v_r \]

10. (b) Time taken to cross the river = \( \frac{d}{v_m \cos \theta} \)
    As he wants to reach at \( B \), then
    \[ v_m \cdot \sin \theta = v_r \]
    or
    \[ \sin \theta = \frac{v_r}{v_m} = \frac{12}{24} = \frac{1}{2} \]
    or
    \[ \theta = 30^\circ \]
    So,
    \[ t = \frac{d}{v_m \cdot \cos \theta} = \frac{10}{24 \times \cos 30^\circ} \]
    \[ = \frac{10 \times 2}{\sqrt{3}} = 5 \times 6\sqrt{3} \text{ s} \]

11. (b) Taking \( x \) and \( y \)-directions as shown in figure.
    Here,
    \[ \mathbf{a}_A = -g \]
    \[ \mathbf{a}_B = -g \]
    \[ u_{Ax} = 60 \cos 30^\circ = 30\sqrt{3} \text{ m/s} \]
    \[ u_{Ay} = 60 \sin 30^\circ = 30 \text{ m/s} \]
    and
    \[ u_{Bx} = -50 \cos \alpha \]
    \[ u_{By} = 50 \sin \alpha \]
    Relative acceleration between the two is zero as \( \mathbf{a}_A = \mathbf{a}_B \).
    Hence, the relative motion between the two is uniform. It can be assumed that \( B \) is at rest and \( A \) is moving with \( u_{AB} \). Hence, the two particles will collide, if \( u_{AB} \) is along \( AB \). This is possible only when \( u_{Ax} = u_{By} \), i.e. component of relative velocity along \( Y \)-axis should be zero.
    \[ \therefore \alpha = \sin^{-1} \left( \frac{3}{5} \right) \]

12. (c) Now, \( |\mathbf{u}_{AB}| = u_{Ax} - u_{Bx} = (30\sqrt{3} + 50 \cos \alpha) \text{ m/s} \)
    \[ = \left( 30\sqrt{3} + 50 \times \frac{3}{5} \right) \text{ m/s} \]
    \[ = \left( 30\sqrt{3} + 40 \right) \text{ m/s} \]
Therefore, time of collision is 
\[ t_{AB} = \frac{|u_{AB}|}{\frac{100}{30\sqrt{3} + 40}} = 1.09 \text{ s} \]

13. (b) Distance of point P from A where collision takes place is
\[ s = \sqrt{\left(u_{A_x} t\right)^2 + \left(u_{A_y} t - \frac{1}{2} g t^2\right)^2} \]
\[ s = \sqrt{\left(30\sqrt{3} \times 1.09\right)^2 + \left(30 \times 10.9 - \frac{1}{2} \times 10 \times 10.9 \times 1.09\right)^2} \]
\[ \Rightarrow s = 62.64 \text{ m.} \]

14. (a) Let us choose the x and y-directions along OB and OA respectively. Then, \( u_x = 10\sqrt{3} \text{ m/s; } u_y = 0 \)
\[ a_x = -g \sin 60^\circ = -5\sqrt{3} \text{ m/s}^2 \]
and \( a_y = -g \cdot \cos 60^\circ = -5 \text{ m/s}^2 \).
At point Q, x-component of velocity is zero. Hence, substituting in \( v_x = u_x + a_x t \)
or \( 0 = 10\sqrt{3} - 5\sqrt{3} t \)
or \( t = \frac{10\sqrt{3}}{5\sqrt{3}} \text{ s.} \)

15. (c) At point Q, \( v_y = u_y + a_y t \)
\[ \therefore v = 0 - \left(5 \cdot 2\right) = -10 \text{ m/s.} \]
Here, negative sign implies that velocity of particle at Q is along negative y-direction.

16. (a) Distance PO = displacement of particle along y-direction
\[ = s_y. \]
Here,
\[ s_y = u_y t + \frac{1}{2} a_y t^2 \]
\[ = -10 \left(5 \cdot 2\right)^2 = -10 \text{ m} \]
\[ \therefore PO = 10 \text{ m.} \]
Therefore, \( h = PO \sin 30^\circ = \left(10 \cdot \frac{1}{2}\right) = 5 \text{ m} \)
\[ \therefore h = 5 \text{ m.} \]

17. (a) Distance OQ = displacement of particle along x-direction
\[ = s_x. \]
Here,
\[ s_x = u_x t + \frac{1}{2} a_x t^2 \]
\[ = \left(10\sqrt{3}\right) - \frac{1}{2} \left(5\sqrt{3}\right) \left(2\right)^2 \]
\[ = 10\sqrt{3} \text{ m.} \]
or \( OQ = 10\sqrt{3} \text{ m.} \)
\[ \therefore \text{POQ} = \sqrt{\left(PO\right)^2 + \left(OQ\right)^2} \]
\[ = \sqrt{\left(10\right)^2 + \left(10\sqrt{3}\right)^2} \]
\[ = \sqrt{100 + 300} = \sqrt{400} = 20 \text{ m.} \]
\[ \therefore \text{POQ} = 20 \text{ m.} \]

18. (b) If we consider elevator at rest, then relative acceleration of the bolt is \( a_{rel} = (9.8 + 1.2), \)
\[ = 11 \text{ m/s}^2 \text{ (downwards).} \]

After 2 s, velocity of lift is \( v = at = (1.2 \cdot 2) = 2.4 \text{ m/s} \)
Therefore, initial velocity of the bolt is also 2.4 m/s and it gets accelerated with relative acceleration 11 m/s² with respect to elevator, initial velocity of bolt is zero and it has to travel 2.7 m with 11 m/s². Thus, time taken can be directly given as
\[ \frac{2s}{a} = \sqrt{\frac{\left(2 \times 2.7\right)^2}{11}} = 0.7 \text{ s} \]

19. (a) Displacement of bolt relative to ground in 0.7s.
\[ s = ut + \frac{1}{2} at^2 \]
\[ s = \left(2.4\right) \left(0.7\right) + \frac{1}{2} \left(-9.8\right) \left(0.7\right)^2 \]
\[ \Rightarrow s = -0.72 \text{ m.} \]

Velocity of bolt will become zero after a time
\[ t_0 = \frac{u}{g} \quad (v = u - gt) \]
\[ = \frac{2.4}{9.8} = 0.245 \text{ s.} \]

Therefore, distance travelled by the bolt = \( s_1 + s_2 \)
\[ \frac{u^2}{2g} + \frac{1}{2} g \left(t - t_0\right)^2 = \left(2.4\right)^2 + \frac{1}{2} \times 9.8 \left(0.7 - 0.24\right)^2 = 1.3 \text{ m.} \]

20. (a) As shown in figure, from equation of motion.
\[ s = s_0 + ut + \frac{1}{2} at^2 \text{ for first gun} \]
\[ x_1 = 5\sqrt{3} t_1 \quad \ldots (i) \]
\[ y_1 = 10 - \frac{1}{2} gt_1^2 \quad \ldots (ii) \]

While for the second gun,
\[ x_2 = 5\sqrt{3} \cos 60^\circ \cdot t_2 \quad \ldots (iii) \]
and
\[ y_2 = 10 + 5\sqrt{3} \sin 60^\circ t_2 - \frac{1}{2} gt_2^2 \quad \ldots (iv) \]

For collision, \( x_1 = x_2 \) and \( y_1 = y_2 \)
i.e., \( 5\sqrt{3} t_1 = 5\sqrt{3} \cdot \frac{1}{2} t_2 \)
and \( 10 - \frac{1}{2} gt_1^2 = 10 + 15 \cdot t_2 - \frac{1}{2} gt_2^2 \)
35

\[ t_2 = 2t_1 \]

and \[ g(t_2^2 - t_1^2) = 15t_2 \]

Which on simplification gives \( t_1 = 1 \) s and \( t_2 = 2 \) s

So, time interval between the firings = \((2 - 1)\) s = 1 s.

21. (b) Substituting the value of \( t_1 = 1 \) s and \( g = 10 \) m/s\(^2\) in Eqs. (i) and (ii), then coordinates of point will be \((5\sqrt{3}, 5)\) m.

22. (a) Due to inertia, ball will share the velocity of platform at the instant of projection. Hence, horizontal components of the ball’s velocity = 3 m/s

Considering the vertical motion,

\[ 10 = 3t + \frac{1}{2} \times 9.8t^2 \quad \text{or} \quad 49 = t^2 + 30t - 100 = 0 \]

or \[ t = \frac{-30 \pm \sqrt{900 + 19600}}{98} \]

Rejecting the negative root, we have \[ t = \frac{-30 + 10\sqrt{250}}{98} = 1.15 \text{ s (approx.)} \]

\[ \therefore \text{The distance } AB = 12 \times 1.15 = 13.80 \text{ m} \]

23. (c) The time taken by the platform to reach the ground \[ \frac{10}{3} = 3.33 \text{ s.} \]

Thus, the time difference = \((3.33 - 1.15)\) = 2.18 s.

24. (a) Let the ball strikes the ground with velocity \( v \), then horizontal component \( v_x = 12 \) m/s

and vertical component \( v_y = (3)^2 + 2 \times 9.8 \times 10 = 143.2 \) m/s.

Then, \( v = \sqrt{v_x^2 + v_y^2} = \sqrt{144 + 205} = \sqrt{349} \) m/s = 18.68 m/s

25. (a) At the highest point of trajectory horizontal force is zero (as \( mg \cos 90^\circ = 0 \)). So, by conservation of linear momentum in horizontal direction.

\[ M \times 50 \cdot \cos \theta + 0 = 4MV. \]

i.e.

\[ V = \left( \frac{50}{4} \right) \cdot \cos \theta \quad \text{...(i)} \]

Now equation of circular motion of the bob at B will be

\[ T = \frac{4MV^2}{l} + 4Mg \cdot \cos 120^\circ \]

But at B, \( T = 0 \) and \( v \neq 0 \) \[ \text{[as angle } > 90^\circ\text{]} \]

So, that the above equation reduces to

\[ v^2 = \frac{2}{2}l = \frac{5}{3}g \quad \text{[as } l = \frac{10}{3} \text{ m]} \quad \text{...(ii)} \]

Now by conservation of mechanical energy between A and B (after collision).

\[ \frac{1}{2}(4M)v^2 = \frac{1}{2}(4M) \cdot v^2 + 4Mg \times \left[ \frac{10}{3} + \frac{10}{3} \sin 30^\circ \right] \]

or \[ \frac{1}{2}v^2 = \frac{1}{2}v^2 + 5g \quad \text{...(iii)} \]

Substituting the values of \( V \) and \( v \) from Eqs. (i) and (ii) in Eq. (iii), we get

\[ \frac{1}{2} \left[ \frac{50}{4} \cdot \cos \theta \right]^2 = \frac{1}{2} \left[ \frac{5}{3}g \right] + 5g \]

i.e.

\[ \frac{25}{2} \cdot \cos \theta = \frac{250}{3} \]

or \[ \frac{25}{2} \cdot \cos \theta = \frac{250}{3} = 10.77 \]

So, \( \cos \theta = 0.86 \)

i.e. \( \theta = 30^\circ \)

26. (a) As initially the bob is at the highest point of trajectory, Horizontal component (X)

\[ \frac{R}{2} = \frac{1}{2} \left( \frac{u^2 \cdot \sin 2\theta}{g} \right) \]

\[ = \frac{50 \times 50 \times \sqrt{3}}{2 \times 10 \times 2} \]

= 108.25 m

Vertical component (Y) = \[ H = \frac{u^2 \sin^2 \theta}{2g} \]

\[ = \frac{50 \times 50 \times 1}{2 \times 10 \times 4} \]

= 31.25 m

INSPIRING ICONS

Rajanikant Nayak, son of a widow daily wage earner, who despite all the obstacles, managed to crack the JEE Advanced. The list of adverse condition that the tribal boy faced is unending, from losing his father at a very young age to having a bed ridden sister. With his mother being the only earning member of the family, it could have been very easy for him to get mowed down by the obstacles and get lost in the crowd of non-descript Denua village under baruan block of Mayurbhanj district in Odisha.

He got an all India ST rank of 245 and is now aiming to join a core stream in any of the Indian Institute of Technology (IIT).
1. In Oersted’s experiment, deflection of magnetic needle produced due to current carrying wire
   (a) remains constant with decrease in current
   (b) increases with increases in current
   (c) first increases then decreases with increases in current
   (d) not found in Oersted’s experiment

2. A current or magnetic field going into the plane of paper is indicated by a ...A... and a current or magnetic field emerging out of plane of paper is indicated by a ...B... . Here, A and B refer to
   (a) cross, dot
   (b) dot, cross
   (c) circle, square
   (d) square, circle

3. Direction of force due to magnetic field on a moving charged particle is
   I. perpendicular to direction of velocity of charged particle.
   II. perpendicular to direction of magnetic field.
   III. parallel to direction of velocity of charged particle.
   IV. parallel to the direction of magnetic field.
   Correct option is
   (a) I and IV
   (b) I and II
   (c) I and III
   (d) III and IV

4. A current carrying wire of area \( A \), length \( l \), number density of charge carriers \( n \) is placed in a region of external magnetic field \( B \), what will be the net force on charge carriers?
   (a) \( nAq(v_B \times B) \)
   (b) \( q(v_B \times B) \)
   (c) \( nAq(v_B \times B) \)
   (d) \( Aq(v_B \times B) \)

5. A straight wire of length 0.5 m carrying a current of 1.2 A placed in a uniform magnetic field of induction 2 T. The magnetic field is perpendicular to the length of the wire. The force on the wire is
   (a) 2.4 N
   (b) 1.2 N
   (c) 3.0 N
   (d) 2.0 N

6. Force acting on a particle does same work only when
   I. force has a component along the direction of motion of particle.
   II. force has a component opposite to direction of motion of particle.
   III. force has a component perpendicular to direction of motion of particle.
   IV. force is not able to move the particle.
   Which of the following statement(s) is/are correct?
   (a) I and II
   (b) II and III
   (c) III and IV
   (d) IV and I

7. Consider a moving charged particle in region of magnetic field. Which of the following are correct?
   I. If \( v \) is parallel to \( B \), then path of particle is spiral.
   II. If \( v \) is perpendicular to \( B \), then path of particle is a circle.
   III. If \( v \) has a component along \( B \), then path of particle is helical.
   IV. If \( v \) is along \( B \), then path of particle is a circle.
   (a) I and II
   (b) II and III
   (c) III and IV
   (d) IV and I

8. An electron and a proton moving on a straight parallel path with same velocity enters a semi-infinite region of uniform magnetic field perpendicular to velocity.
Which of these are correct?
I. They will never come out of magnetic field region.
II. They will come out travelling along parallel paths.
III. They will come out same time.
IV. They will come out at different times.
(a) I and II
(b) II and III
(c) II and IV
(d) I and IV

9. A velocity selector, (a region of perpendicular electric and magnetic field)
I. Allows charged particles to pass straight when \( v = E / B \).
II. Deflects particle in a direction perpendicular to both \( v \) and \( B \), where, \( v > E / B \).
III. Deflects particle in the direction of electric field when, \( v < E / B \).
IV. Deflects all particles in a direction perpendicular to both \( E \) and \( B \).
(a) I, III and IV
(b) II, III and IV
(c) I, II and III
(d) I, II and IV

10. For the sketch of cyclotron given match the name of parts of cyclotron with their levels.

<table>
<thead>
<tr>
<th>Column I (Name)</th>
<th>Column II (Labels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Metal dees</td>
<td>1</td>
</tr>
<tr>
<td>B. Oscillator</td>
<td>2</td>
</tr>
<tr>
<td>C. Exit port</td>
<td>3</td>
</tr>
<tr>
<td>D. Magnetic field out of the paper</td>
<td>4</td>
</tr>
<tr>
<td>E. Deflection plate</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Codes</th>
<th>A B C D E</th>
<th>A B C D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1 2 3 4 5</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>(c)</td>
<td>3 1 2 4 5</td>
<td>2 3 4 1</td>
</tr>
</tbody>
</table>

11. Similarities of Biot-Savart’s law and Coulomb’s law for the electrostatics,
I. both are long range and inversely proportional to the square of distance from the source to point of interest.
II. both are linear in source.
III. both are produced by scalar sources.
IV. both follow principles of superposition.
(a) I, II, III
(b) II, III, IV
(c) I, II, IV
(d) I, III, IV

12. An element \( \Delta I = \Delta x \hat{i} \) is placed at the origin and carries a current \( I = 10 \, \text{A} \).

13. Match the following columns.

<table>
<thead>
<tr>
<th>Column I (Name)</th>
<th>Column II (Labels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Lorentz force</td>
<td>1. ( \int E \cdot dA = \frac{q}{\varepsilon_0} )</td>
</tr>
<tr>
<td>B. Gauss’s law</td>
<td>2. ( dB = \frac{\mu_0}{4\pi} \frac{idr \times r}{r^3} )</td>
</tr>
<tr>
<td>C. Biot-Savart’s law</td>
<td>3. ( F = q [E + (v \times B)] )</td>
</tr>
<tr>
<td>D. Coulomb’s law</td>
<td>4. ( F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} )</td>
</tr>
</tbody>
</table>

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<tr>
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<tr>
<td>(a)</td>
<td>3 1 2 4</td>
<td>1 2 4 3</td>
</tr>
<tr>
<td>(c)</td>
<td>2 3 1 4</td>
<td>4 1 2 3</td>
</tr>
</tbody>
</table>

14. Which of the following figures correctly depicts the direction of magnetic field of a current carrying coil?

15. A straight wire carrying a current of 12 A is bent into a semicircular loop of radius 2.0 cm as shown in figure. What will be magnetic field at the centre of loop?

(a) \( B = 1.9 \times 10^{-4} \, \text{T} \)
(b) \( B = 1.9 \times 10^{4} \, \text{T} \)
(c) \( B = 1.9 \times 10^{-4} \, \text{T} \), \( \Phi \)
(d) \( B = 1.9 \times 10^{4} \, \text{T} \), \( \Phi \)
16. A straight long wire carries a current of 35 A. Its magnitude of magnetic field at a distance of 0.20 m from the wire is
(a) $3.5 \times 10^{-5}$ T
(b) $3.5 \times 10^{-5}$ T
(c) 3.5 T
(d) 7 T

17. Two concentric circular coils x and y of radii 16 cm and 10 cm lie in same vertical plane containing north to south direction. Coil x has 20 turns, coil y has 25 turns and current in coil x is 16 A whereas in coil y is 18A. Current in x is anti-clockwise and in y is clockwise. For an observer facing west and looking at coils, magnetic field at the centre of assembly of coils is
(a) $1.57 \times 10^{-3}$ T towards east
(b) $1.57 \times 10^{-3}$ T towards west
(c) $1.57 \times 10^{-3}$ T towards north
(d) $1.57 \times 10^{-3}$ T towards south

18. For a cylindrical conductor of radius $a$, which of the following graphs shows a correct relationship of $B$ versus $r$?

19. A current $I$ flows along the length of an infinitely long straight thin walled pipe, then
(a) the magnetic field is zero only at axis of the pipe
(b) the magnetic field is different at different points inside the pipe
(c) the magnetic field at any point inside the pipe is zero
(d) the magnetic field at all points inside the pipe is the same but not zero

20. Which is a correct figure to display the magnetic field lines due to a solenoid?
(a) Cross-section of wires
(b) Cross-section of wires
(c) Cross-section of wires
(d) Cross-section of wires

21. Which one is a correct figure to represent magnetic bottle in a plasma fusion experiment?

22. A solenoid of length 1 m and 30 cm diameters has five layers of windings of 850 turns each and carries a current of 5A. Calculate the magnetic flux for a cross-section of the solenoid as the centre of the solenoid.
(a) $2.45 \times 10^{-5}$ Wb
(b) $4 \times 10^{-3}$ Wb
(c) $1.89 \times 10^{-5}$ Wb
(d) $3 \times 10^{-2}$ Wb

23. Which one is a correct shape for current carrying wires?
(a) A and B
(b) B and C
(c) C and D
(d) D and A

24. If two parallel current carrying conductors placed one metre apart in vacuum are placed such that each carries $I$ current, then there is a force of
(a) $2 \times 10^{-7}$ N per metre of length
(b) $2 \times 10^{-7}$ N per metre of length
(c) $9 \times 10^{8}$ N per metre of length
(d) $9 \times 10^{-8}$ N per metre of length
25. For a current carrying wire loop of N turns, placed in region of a uniform magnetic field \( B \), match column I and column II.

<table>
<thead>
<tr>
<th>Column I</th>
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</tr>
</thead>
<tbody>
<tr>
<td>A. Torque on loop</td>
<td>1. ( mB )</td>
</tr>
<tr>
<td>B. Torque on loop when ( m ) is either parallel or anti-parallel to ( B )</td>
<td>2. ( NIA )</td>
</tr>
<tr>
<td>C. Magnetic moment of loop</td>
<td>3. Zero</td>
</tr>
<tr>
<td>D. Torque on loop when ( m ) is perpendicular to ( B )</td>
<td>4. ( m \times B )</td>
</tr>
</tbody>
</table>

Codes

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<td>4</td>
</tr>
<tr>
<td>(c)</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>(d)</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

26. A current carrying circular loop lies on a smooth horizontal plane.

(a) If a uniform magnetic field is set up parallel to plane of loop, it turns about the vertical axis.
(b) If a uniform magnetic field is set up perpendicular to plane of loop it turns about the vertical axis.
(c) If a uniform magnetic field is set up perpendicularly towards loop turns up about the vertical axis.
(d) Torque produced is always in the plane perpendicular to \( B \).

27. In a moving coil galvanometer of coil of \( N \) turns of area \( A \) having a spring of stiffness \( k \). If coil is deflected by some angle \( \phi \) due to flow of \( I \) current in uniform radial magnetic field \( B \), then

(a) \( \phi = \left( \frac{NAB}{k} \right) I \)
(b) \( \phi = \left( \frac{kA}{BNA} \right) I \)
(c) \( \phi = \left( \frac{kA}{BN} \right) I \)
(d) \( \phi = \left( \frac{BN}{kA} \right) I \)

28. The galvanometer cannot as such be used as ammeter to measure the value of current in a given circuit. The following reasons are:

I. galvanometer gives full scale deflection for a small current.
II. galvanometer has a large resistance.
III. a linear scale cannot be designed so that \( I \propto \phi \).
IV. a galvanometer can give in accurate values.

(a) I and IV
(b) II and III
(c) I and II
(d) III and IV

29. To convert a galvanometer into a voltmeter

(a) a low resistance in parallel is used
(b) a low resistance in series is used
(c) a high resistance in series is used
(d) a high resistance in parallel is used

30. To increase the current sensitivity of a moving coil galvanometer, we should decrease

(a) a strength of magnet
(b) torsional constant of spring
(c) number of turns in coil
(d) area of coil

Correct Answers along with NCERT Textbook Reference

1. (b) Introduction, Page No. 132
2. (a) Introduction, Page No. 133
3. (b) Magnetic force, Page No. 134
4. (a) Magnetic field, Lorentz force, Page No. 134
5. (b) Magnetic force on a current carrying conductor, Page No. 136-137
6. (a) Motion in a magnetic field, Page No. 137
7. (b) Motion in a magnetic field, Page No. 138
8. (c) Motion in a magnetic field, Page No. 138
9. (c) Motion in combined electric and magnetic fields, Page No. 140
10. (d) Motion is combined electric and magnetic fields, Page No. 141
11. (c) Magnetic field due to a current element, Biot-Savart law, Page No. 143
12. (a) Magnetic field due to a current element, Biot-Savart law, Page No. 144
13. (a) Magnetic field due to a current element, Biot-Savart law, Page No. 144
14. (a) Magnetic field on the axis of a circular current loop, Page No. 145
15. (c) Magnetic field on the axis of a circular current loop, Page No. 145
16. (a) Magnetic field on the axis of a circular current loop, Page No. 146
17. (b) Magnetic field on the axis of a circular current loop, Page No. 147
18. (c) Ampere’s circuital law, Page No. 147
19. (c) Ampere’s circuital law, Page No. 148
20. (d) The solenoid, Page No. 151
21. (b) The solenoid, Page No. 152
22. (c) The toroid, Page No. 154
23. (b) Force between two parallel currents, the ampere, Page No. 154
24. (a) Force between two parallel currents, the ampere, Page No. 155
25. (c) Torque on current loop, magnetic dipole, Page No. 157
26. (d) Circular current loop as a magnetic dipole, Page No. 161
27. (a) The moving coil galvanometer, Page No. 164
28. (c) The moving coil galvanometer, Page No. 164
29. (c) The moving coil galvanometer, Page No. 165
30. (b) The moving coil galvanometer, Page No. 166
Electric current It is defined as the time rate of flow of electric charge through a cross-section of the conductor. Instantaneous value of current at any instant is given by

\[ I = \lim_{\Delta t \to 0} \left( \frac{\Delta Q}{\Delta t} \right) = \frac{dQ}{dt} \]

The SI unit of electric current is ampere (A).

Drift velocity It is defined as the average velocity with which free electrons get drifted towards the positive end of the conductor under the influence of an external electric field, i.e.

\[ v_d = -\frac{eE}{m} \]

where, \( e \) is the charge on electron, \( m \) is the mass, \( E \) is the electric field applied and \( \tau \) is the average time of relaxation.

Mobility It is defined as the drift velocity \( (v_d) \) per unit electric field applied i.e.

\[ \mu = \frac{v_d}{E} \]

Relation between current and drift velocity/mobility \( I = n e A v_d = n e A \mu E \), where \( n \) is the electron density or number of electrons per unit volume of the conductor and \( A \) is the area of cross-section of the conductor.

Ohm’s law It states that the current \( (I) \) flowing through a conductor is directly proportional to the potential difference \( (V) \) across the end of the conductor, provided physical conditions of the conductor such as temperature, mechanical strain, etc., are kept constant, i.e. \( V \propto I \) or \( V = IR \) where, \( R \) is known as the resistance of the conductor.

Specific resistance or electrical resistivity It is defined as the resistance of unit length and unit area of cross-section of the conductor i.e.

\[ \rho = \frac{m}{n e^2 \tau} \]

The SI unit of resistivity is \( \Omega \cdot m \).

Current density \( (J) \) at a point is defined as the amount of current flowing per unit area of cross-section of the conductor, provided the area is held in a direction normal to the current.

\[ J = \frac{I}{A} \]

The SI unit of current density is \( \text{Am}^{-2} \).

Electrical conductivity \( (\sigma) \) of a conductor is the inverse of its resistivity \( (\rho) \), i.e. \( \sigma = 1/\rho \). The SI unit of \( \sigma \) is \( \Omega^{-1} \cdot \text{m}^{-1} \) or \( \text{Sm}^{-1} \).

Effect of temperature on resistance \( \bullet \) The resistance of a metal conductor at a temperature \( T \) is given by \( R_T = R_0 (1 + \alpha T) \), where, \( R_0 \) is the resistance of a conductor at \( 0^\circ \text{C} \) and \( \alpha \) is the temperature coefficient of resistance.

For metals, \( \alpha \) is the positive, i.e. resistance increases with rise in temperature.

For semiconductor and insulators, \( \alpha \) is negative, i.e. resistance decreases with rise in temperature.

For alloys like manganin, eureka and constantan, the value of \( \alpha \) is very small as compared to that of conductors.

If \( R_1 \) and \( R_2 \) are the resistances of the same conductor at temperatures \( t_1 \) \( ^\circ \text{C} \) and \( t_2 \) \( ^\circ \text{C} \), then

\[ R_{t_2} = R_{t_1} [1 + \alpha (t_2 - t_1)] \]

Non-Ohmic conductors Conductors which do not obey Ohm’s law which may represent the following points

- The straight line of \( V-I \) graph does not pass through the origin.
- \( V-I \) relationship is non-linear.
- \( V-I \) relationship depends on the sign of \( V \) for the same absolute value of \( V \).
- \( V-I \) relationship is non-unique.

Superconductors The certain metals and alloys substances which completely lose their resistivity when they are supercooled below a certain temperature e.g. mercury at temperature 4.2K lead at 7.25K becomes superconductors.

Meissner effect The exclusion of the magnetic flux from a superconducting material when it is cooled to a temperature below the critical temperature in a magnetic field is called as Meissner effect.

Series grouping of resistances

- In series, same current \( I \) flows through all the resistances.
- The potential difference across the combination is distributed across various resistors in the direct ratio of their resistances i.e.

\[ V = V_1 + V_2 + V_3 + \ldots \]

- Total equivalent resistance in series grouping of resistances \( R_e \) is equal to the sum of individual resistance.

\[ R_e = R_1 + R_2 + R_3 + \ldots \]

Parallel grouping of resistances

(a) In parallel grouping of resistances, same potential difference \( V \) appears across each resistance.

(b) The current is distributed among various resistor in the inverse ratio of their resistances.

Thus, \( I = I_1 + I_2 + I_3 + \ldots \)

Total equivalent resistance in parallel grouping of resistance \( R_p \) is given by

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

Colour code for carbon resistors The number attached from 0 to 9, 10, 100, 1000, 10000, 100000, ... 100000000... etc., are the colour codes for carbon resistors with which the carbon resistors are marked.

- Black-0, Brown-1, Red-2, Orange-3, Yellow-4, Green-5, Blue-6, Violet-7, Grey-8, White-9, Gold-10, Silver-11.

The straight line of \( V-I \) graph does not pass through the origin and \( (V/I) \) is constant and also \( (V/I) \) is independent of temperature.

Current reaching a junction if taken positive, then the current leaving the junction is taken negative. This law supports the concept that moving charges are not accumulated at a junction i.e. the law of conservation of charges.

- First rule In a closed loop, the algebraic sum of all the potential difference is zero, i.e. \( \Sigma V = 0 \). This rule supports the law of conservation of energy.

Kirchhoff’s law Kirchhoff gave two laws which are as follows;

First rule The algebraic sum of the current meeting at a junction is zero, i.e.

\[ \Sigma I = 0 \]

Wheatstone bridge principle It states that if four resistances \( P, Q, R, S \) are arranged to form a bridge as shown in figure, on pressing battery key \( K_1 \), first and then galvanometer key \( K_2 \), if the galvanometer shows no deflection, then the bridge is balanced, i.e.

\[ \frac{P}{Q} = \frac{R}{S} \]
**COMPARATIVE ANALYSIS BETWEEN MEASURING INSTRUMENTS**

<table>
<thead>
<tr>
<th>Galvanometer</th>
<th>Ammeter</th>
<th>Voltmeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>It detects current and connected in series with the circuit through which current is to be detected.</td>
<td>It measures current and connected in series with the element through which current is to be measured.</td>
<td>It measures potential difference and connected in parallel with the element across which potential difference is to be measured.</td>
</tr>
<tr>
<td>The resistance of galvanometer is more than that of ammeter but less than that of voltmeter.</td>
<td>The reading of ammeter is lesser than true value.</td>
<td>The reading of voltmeter is lesser than true value.</td>
</tr>
<tr>
<td>Smaller the resistance of ammeter more accurate is its reading.</td>
<td>Greater the resistance of voltmeter more accurate is its reading.</td>
<td>The resistance of ideal galvanometer is zero.</td>
</tr>
</tbody>
</table>
| The resistance of ideal ammeter is zero. | The resistance of ideal voltmeter is infinity. | *The practical application of Wheatstone bridge principle is in meter bridge or slide wire bridge and post office box which are used to find the unknown resistance or specific resistance of the given metallic wire.*

**Potentiometer**
- It is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform across of cross-section and a constant current is flowing through it i.e. \( V \propto l \) or \( V = kl \), where \( K \) is called potential gradient.
- **Applications of Potentiometer**
  - **Measurement of potential drop across a resistor** (\( R \)) Let potential drop across a resistor \( R \) is \( V \).
  - **Comparison of emf’s of two batteries**
    - Let \( E_1 \) and \( E_2 \) be emf’s of batteries 1 and 2 respectively, \( I_1 = \text{balancing length (AX)} \) when switches 1 and 2 are connected.
    - \( I_2 = \text{balancing length (AX)} \) when switches 2 and 3 are connected.

**Electric energy**
- Total work done (or energy supplied) by the source of emf in maintaining the electric current in the circuit for a given time is called electric energy consumed in the circuit.
- The electric energy consumed in kWh is given by
  \[
  W = \frac{V}{1000} \text{ (in volt)} \times \frac{I}{1000} \text{ (in amp)} \times t \text{ (in hour)}
  \]

**Heating effect of current**
- The phenomenon in which heat energy is produced in a conductor due to the flow of electric current is known as heating effect of current.
- Heat energy produced is given by
  \[
  H = W = I^2 RT
  \]
  (in joule)
  \[
  H = \frac{1}{1.18} \frac{l^2 I^2 RT}{m^2 q}
  \]
  (in calories)
  where, \( I = \text{current flowing}, \) \( R = \text{resistance} \) and \( t = \text{time taken}. \)

**First law of electrolysis**
- According to this law, the mass of a substance deposited or liberated at an electrode is directly proportional to the quantity of electricity i.e. charge through the electrolyte.
  \[
  m \propto q \text{ or } m = qZ
  \]
  where, \( Z \) is called electrochemical equivalent.

**Second law of electrolysis**
- According to this law, when same charge is passed through different electrolytes, then mass substances deposited or liberated directly proportional to their chemical equivalent
  \[
  \frac{M_1}{M_2} = \frac{E_1}{E_2} \text{ if } q_1 = q_2.
  \]

**Relation between \( E, Z \) and Faraday’s constant**
- Relation between thermoemotivc force (\( E \)), electrochemical equivalent (\( Z \)) and Faraday’s constant is given by
  \[
  \frac{m_1}{m_2} = \frac{Z_1}{Z_2}
  \]
  and
  \[
  \frac{m_1}{m_2} = \frac{E_1}{E_2}
  \]
  or
  \[
  \frac{E_1}{Z_1} = \frac{E_2}{Z_2} = \text{constant}.
  \]
  Therefore, \( \frac{E}{Z} = F \), where \( F \) is called Faraday’s constant.

**Applications of thermolectric effect**
- **To detect heat radiation** A thermo pile is a sensitive instrument used for detection of heat radiation and measurement of their intensity. It is based upon Seeback effect.
- **Thermoelectric refrigerator** The working of thermoelectric refrigerator is based on Peltier effect.
- **Thermoelectric generator** Thermocouple can be used to generate electric power using Seeback effect in remote areas.
CIRCULAR MOTION

Coverage of special topic for JEE Advanced

Circular motion is a curvilinear motion in which path of particle is a circle. Few examples are:

- Motion of any point on a rotating disc or ring.
- Motion of electron in a Bohr’s orbit.
- Motion of a wheel around its axis.
- Motion of earth around the sun as an approximation. (Approximation because orbit is necessarily elliptical).

A body in circular motion essentially have an angular velocity and it may have an angular acceleration.

**Angular Velocity**

For arc length $S$,

$$S = R\theta$$  \hspace{1cm} (i)

Differentiating Eq (i) with respect to $t$, we have

$$\frac{dS}{dt} = R \cdot \frac{d\theta}{dt} \Rightarrow v = R\omega$$

where,

$$\frac{dS}{dt} = v = \text{tangential velocity}$$

and

$$\frac{d\theta}{dt} = \omega = \text{angular velocity}.$$

Angular velocity is a vector with unit radians per second (In SI system it is $\text{S}^{-1}$) and dimension $[\text{T}^{-1}]$.

**Angular Acceleration**

Angular acceleration is defined as

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt} \left(\frac{v}{R}\right) = \frac{a_T}{R}$$

$
\alpha$ is a vector with units $\text{rad/s}^2$ and dimension $[\text{T}^{-2}]$.

**Example**

**PROBLEM BASED ON ANGULAR SPEED**

A traveller reaches a city and she observes there quickest sunrise and sunsets and length of day is nearly equal in all seasons. Find

(a) where on earth this city is located?

(b) what is the magnitude of velocity of this person relative to an axis through earth’s centre?

(c) what is the acceleration of this person?
Sol. (a) Step I Find location of the city
City must be located over equator because sun moves nearly perpendicular to the horizon most of the year resulting in quickest, sunrise and sunset.
E.g. Namnyki (Kenya) or (Macapa Brazil)

Step II Use, \( \mathbf{v} = R\omega \)
(b) Using \( \mathbf{v} = R\omega \)
we have,
\[
\mathbf{v} = 6400 \text{ km} \times \left( \frac{2\pi}{24 \text{ h}} \right)
= 1675 \text{ m/s}
= 465 \text{ m/s} \text{ (A quite high speed)}
\]

Step III Find acceleration
(c) Acceleration
\[
a = \frac{\mathbf{v}^2}{R} = R\omega^2
= 6400 \text{ km} \times \left( \frac{2\pi \text{ rad}}{24 \text{ h}} \right)^2
= \frac{6400 \times (2 \times 3.14)^2}{(24)^2}
= 438 \text{ km} \text{ k}^2
= 34 \text{ m/s}^2
\]

Classification of Circular Motion
A circular motion may be uniform or non-uniform according to value of \( \alpha \).
- When \( \alpha = 0 \), motion is uniform circular motion and \( \omega \) is a constant.
- When \( \alpha \neq 0 \), motion is non-uniform circular motion and \( \omega \) is a variable.

Uniform Circular Motion
For a particle \( A \) of mass \( m \), moving around a circular path of radius \( R \), with a constant angular speed \( \omega \),
\[
\begin{align*}
\mathbf{v} & = \omega \times \mathbf{r} \\
\mathbf{a} & = -\omega^2 \mathbf{r}
\end{align*}
\]

Taking centre at origin, we have
(i) Position (or position vector) of particle (at instant \( t \))
\[
\mathbf{OA} = \mathbf{r} = x\hat{i} + y\hat{j}
\]
where, \( x = R \cos \omega t \)
and \( y = R \sin \omega t \)
\[
\therefore \mathbf{r} = (R \cos \omega t)\hat{i} + (R \sin \omega t)\hat{j}
\]

(ii) Velocity of particle is
\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} = -\omega R \sin \omega t \hat{i} + \omega R \cos \omega t \hat{j}
\]

So, particle has velocity
\[
\mathbf{v} = v_x \hat{i} + v_y \hat{j}
\]
\[
\Rightarrow |v_x| = \omega R \sin \omega t \text{ and } |v_y| = \omega R \cos \omega t
\]
and
\[
|\mathbf{v}| = \text{magnitude of velocity}
= \sqrt{v_x^2 + v_y^2}
= \omega R
\]

In vector form,
\[
\mathbf{v} = \omega \times \mathbf{r}
\]

So,
\[
|\mathbf{v}| = \omega R \sin \theta = \omega R
\]
When origin \( O \) and centre of circle \( C \) coincides, \( \theta = 90\degree \) and sin \( \theta = 1 \)
\[
\therefore |\mathbf{v}| = \omega R
\]

(iii) Acceleration of particle is
\[
\mathbf{a}_N = \frac{d\mathbf{v}}{dt} = a_x \hat{i} + a_y \hat{j}
\]
\[
= -\omega^2 R \cos \omega t \hat{i} - \omega^2 R \sin \omega t \hat{j}
\]
\[
\Rightarrow |\mathbf{a}_N| = \text{magnitude of acceleration}
= \sqrt{a_x^2 + a_y^2}
= \omega^2 R
= \frac{v^2}{R} \quad (\because v = R\omega)
\]

So, there is an acceleration \( \mathbf{a}_N \) of particle directed towards centre, called centrifugal acceleration (or normal acceleration).
(iv) Force on the particle in uniform circular motion is given by

\[ \mathbf{F} = \text{centripetal force} = \text{mass} \times \text{acceleration} = \frac{mv^2}{R} \]

This force is perpendicular to the direction of velocity of particle.

(v) Work done by centripetal force is given by

\[ W = (\mathbf{F} \cdot \mathbf{v}) \Delta t = 0 \]

\( \mathbf{F} \cdot \mathbf{v} = \text{power} = \Delta t = \text{time interval} \quad \therefore \mathbf{A} \cdot \mathbf{B} = 0 \text{ when } \mathbf{A} \perp \mathbf{B} \)

So, no work is done by the centripetal force on the particle.

(vi) Kinetic energy of rotating particle is

\[ K = \frac{1}{2} \frac{mv^2}{R} = \frac{1}{2} mR^2 \omega^2 \quad (v = R \omega) \]

KE of rotating particle is constant as work done by force is zero.

(vii) As \( \mathbf{a}_r = \frac{d\mathbf{v}}{dt} \)

We have

\[ \mathbf{a}_r = \frac{d}{dt} (\mathbf{v} \times \mathbf{r}) = \mathbf{\omega} \times \mathbf{v} \quad \therefore \mathbf{a}_r = \mathbf{\omega} \times \mathbf{v} \]

(b) magnitude of tangential velocity of ball.

(c) find position vector of ball at \( t = 0 \)

(Take, particle at \( \theta = 0 \), at \( t = 0 \) and it moves in \( XOY \)-plane).

(d) find velocity vector of ball at, \( t = \frac{\pi}{12} \) s.

(e) find acceleration vector of ball at, \( t = \frac{\pi}{12} \) s.

So. \hspace{1cm} \text{Step I} \hspace{0.5cm} \text{Calculate angular speed of ball using}

\[ \omega = \frac{2\pi}{T} \]

or \[ \omega = \frac{2\pi}{T} = 2 \text{ radian per second.} \]

\[ \Rightarrow \omega = 2 \text{ rad/s} \]

\text{Step II} \hspace{0.5cm} \text{Tangential velocity is given by, } \mathbf{v} = \omega \mathbf{r}

\[ \mathbf{v} = \omega \mathbf{r} \]

\[ = 2 \text{ rad/s} \times 2.5 \text{ m} \]

\[ = 5 \text{ m/s} \]

\text{Step III} \hspace{0.5cm} \text{Calculate angle turned in } \frac{\pi}{12} \text{ second and then find } x \text{ and } \ y\text{-coordinates of particle using } x = R \cos \omega t \text{ and } y = R \sin \omega t . \]

\[ \text{(c) Angle turned in seconds, } \theta = \omega t \]

\[ \Rightarrow \theta = 2 \text{ rad/s} \times \frac{\pi}{12} \text{ s} \]

\[ = \frac{\pi}{6} \text{ radians} = 30^\circ \]

Coordinates of point \( P \) are

\[ x = R \cos \theta = 2.5 \times \cos 30^\circ \]

\[ = 2.5 \times \frac{\sqrt{3}}{2} \text{ m} \]

\[ y = R \sin \theta = 2.5 \times \frac{1}{2} \text{ m} \]

Hence, position of particle at \( t = \frac{\pi}{12} \) s is

\[ \mathbf{OP} = x\mathbf{i} + y\mathbf{j} \]

\[ \Rightarrow r = 2.5 \times \frac{\sqrt{3}}{2} + 2.5 \times \frac{1}{2} \mathbf{j} \]

\[ \Rightarrow r = 2.5 \left( \frac{\sqrt{3}}{2} + \frac{1}{2} \right) \]

\text{Step IV} \hspace{0.5cm} \text{Now calculate velocity components using}

\[ v_x = \frac{dx}{dt} = - R \omega \sin \omega t \]

\[ = - R \omega \sin \theta \]

and \[ v_y = \frac{dy}{dt} = R \omega \cos \omega t \]

\[ = R \omega \cos \theta \]

\text{Example 2}

PROBLEM BASED ON UNIFORM CIRCULAR MOTION

A metal ball is moving in a circular groove of radius 2.5 m.

If speed of ball is constant and it completes 1 round in \( \pi \) second, then calculate

(a) angular speed of ball.
(d) \[ v_y = -R\omega \sin \theta \]
\[ = -2.5 \times 2 \times \sin 30^\circ \]
\[ = 2.5 \text{ ms}^{-1} \]
\[ v_y = R\omega \cos \theta \]
\[ = 2.5 \times 2 \times \cos 30^\circ \]
\[ = 2.5 \times 2 \times \frac{\sqrt{3}}{2} \]
\[ = 2.5 \sqrt{3} \text{ ms}^{-1} \]

So, velocity vector of particle at \( t = \frac{\pi}{12} \) s is
\[ v = v_x \hat{i} + v_y \hat{j} \]
\[ \Rightarrow v = -2.5\hat{i} + 2.5\sqrt{3}\hat{j} \]
\[ \Rightarrow v = 2.5 (-\hat{i} + \sqrt{3}\hat{j}) \text{ ms}^{-1} \]

**Step V** Find components of acceleration.
\[ a_x = \frac{dv_x}{dt} = -\omega^2 R \cos \theta \]
and \[ a_y = \frac{dv_y}{dt} = -\omega^2 R \sin \theta \]
(e) Components of acceleration are
\[ a_x = -2 \times 2.5 \times \cos 30^\circ \]
\[ = -2 \times 2.5 \times \frac{\sqrt{3}}{2} \]
\[ = -2.5 \times \sqrt{3} \text{ ms}^{-2} \]
and \[ a_y = -2 \times 2.5 \times \frac{1}{2} \]
\[ = -2.5 \text{ ms}^{-2} \]
So, \[ a = -2.5(\sqrt{3} \hat{i} + \hat{j}) \text{ ms}^{-2} \]

**Non-Uniform Circular Motion**

In general case of non-uniform circular motion, both the magnitude and the direction of velocity changes.

(i) Tangential acceleration is the rate of change of magnitude of tangential velocity.
\[ a_T = \text{tangential acceleration} \]
\[ = \frac{d}{dt} (v) = \frac{d}{dt} R\omega = R \frac{d\omega}{dt} \]

(ii) Angular acceleration is the rate of change of angular velocity.
\[ \alpha = \text{Angular acceleration} = \frac{d\omega}{dt} \]

(iii) Total acceleration of the particle is resultant of tangential and normal (centripetal) acceleration.
\[ \therefore a = a_T + a_N \]
where, \( a_N = \omega^2 R \) and \( a_T = R\alpha \)

(iv) Equations of motion in case of a rotating body having constant angular acceleration are
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 = \omega_0^2 + 2\alpha \theta \]
and \[ \theta = \frac{\alpha}{2} t^2 + \frac{\omega_0}{2} t \]

(v) For a body moving over a circular path with variable angular acceleration, we have
\[ \alpha = \frac{d\omega}{dt} \Rightarrow \int_{\omega_0}^{\omega} d\omega = \int_{t_0}^{t} \alpha \cdot dt \]
Also, \[ \omega = \frac{d\theta}{dt} \Rightarrow \int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} \omega \cdot dt \]

**Example**

**PROBLEM BASED ON NON-UNIFORM CIRCULAR MOTION WITH VARIABLE ACCELERATION**

A body initially at rest \( (\theta = 0 \text{ and } \omega = 0 \text{ at } t = 0) \) and is accelerated over a circular path of radius \( R = 1.3 \text{ m} \) as per the relation, \( \alpha = 120t^2 - 48t + 16 \text{ rad / s}^2 \). Find

(a) angular speed of body.
(b) position of body at instant \( t \).
(c) tangential acceleration of body.
(d) centripetal acceleration of body.

**Sol.**

**Step I** Calculate angular speed using,
\[ \alpha = \frac{d\omega}{dt} \Rightarrow \omega = \int \alpha \cdot dt \]

(a) \[ \therefore \omega = \int (120t^2 - 48t + 16) dt = \int \frac{120t^3}{3} - \frac{48t^2}{2} + 16t + c \text{ rad/s} \]
\[ \text{or } \omega = 40t^3 - 24t^2 + 16t + c \]
At, \( t = 0, \omega = 0 \) \( (: c = 0) \)
Hence, \( \omega = 40t^3 - 24t^2 + 16t \text{ rad/s} \)

**Step II** Calculate position using,
\[ \frac{d\theta}{dt} = \omega \]

(b) \[ \therefore \theta = \int_0^t \omega dt = \int_0^t (40t^3 - 24t^2 + 16t) dt \]
\[ \therefore \theta = 10t^4 - 8t^3 + 8t^2 \text{ rad} \]

**Step III** Use \( v = R\omega \) and \( a_T = \frac{dv}{dt} \) to find tangential acceleration.

(c) So, \[ v = R\omega = 1.3(40t^3 - 24t^2 + 16t) \text{ ms}^{-1} \]
and \( a_T = \text{tangential acceleration} \)
\[ = \frac{d}{dt} (1.3(40t^3 - 24t^2 + 16t)) \]
\[ = 156t^2 - 62.4t + 20.8 \text{ ms}^{-2} \]

**Step IV** Use \( a_c = \frac{v^2}{R} \) or \( R\omega^2 \)

(d) \[ \therefore a_c = \text{centripetal acceleration} \]
\[ = R\omega^2 \]
\[ = 1.3(40t^3 - 24t^2 + 16t)^2 \]
\[ = 83.2t^2(5t^2 - 3t + 2)^2 \text{ ms}^{-2} \]
Applications of Circular Motion Analysis in Solving Problems of Laws of Motion

Circular motion is a very common occurrence practically. Few of them are:

(I) Conical Pendulum

Bob of conical pendulum has zero vertical acceleration and non-zero horizontal acceleration.

From Newton’s second law,

\[ \Sigma F_x = 0 \]
\[ \Rightarrow F \cos \beta = m \frac{a_x}{g} \]
and
\[ \Sigma F_y = 0 \]
\[ \Rightarrow F \sin \beta = m \frac{a_y}{g} \]
\[ \Rightarrow a_y = \frac{4\pi^2 R}{T^2} \]
\[ \Rightarrow a_N = \frac{4\pi^2 R}{T^2} \]
\[ \Rightarrow \frac{4\pi^2 R}{T^2} \quad \begin{align*}
\therefore T &= \frac{2\pi}{v} \\
\therefore a_N &= \frac{v^2}{R}
\end{align*} \]

So, time period of revolution,
\[ T = 2\pi \sqrt{\frac{R}{g \tan \beta}} \]
\[ \Rightarrow T = 2\pi \sqrt{\frac{L \cos \beta}{g}} \]
\[ \therefore R = L \sin \beta \]

Also, linear velocity of bob is
\[ v = \omega R \]
\[ = \frac{2\pi}{T} \cdot R = \sqrt{gR \cdot \tan \beta} \]

Tension in string is
\[ F = \frac{mg \cos \beta}{\cos \beta} = \frac{mgL}{\sqrt{L^2 - R^2}} \]

(Example 4) To calculate angular velocity of a ball moving in a horizontal circle

A ball of mass (m) 0.5 kg is attached to the end of a string having length (l) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is

(a) 9  (b) 18  (c) 27  (d) 36

Sol. (d) Key Concept Components of tension in the string will balance the weight as well as provide centripetal force.

(Example 5) To find elongation in a spring, supporting a horizontal circulating mass.

A bob of mass m is connected to a spring of spring constant k and a string which is fixed to a ceiling as shown in figure. Original length of the spring is L and the bob rotates in a horizontal circle centred at the fixed rod. Assuming the surfaces are smooth and spring remains horizontal, then elongation in the spring is

\[ \begin{align*}
(a) & \quad \frac{(mg - ma^2L) \tan \theta}{ma^2 + k} \\
(b) & \quad \frac{mg \tan \theta + ma^2L}{ma^2 - k} \\
(c) & \quad \frac{ma^2L}{k} \\
(d) & \quad \frac{9 \tan \theta - \omega^2L}{\omega^2 - k/m}
\end{align*} \]
**Sol.**  **Step I** Indicate the forces acting on the bob, during the circular motion.

Let elongation in the spring is $x$. Therefore, outward horizontal centrifugal force on the bob is $ma^2(L + x)$.

**Step II** Balance vertical forces acting on the bob

$$T \cos \theta = mg$$  \hspace{1cm} \cdots (i)

**Step III** Balance horizontal forces acting on the bob

$$T \sin \theta + kx = ma^2(L + x)$$

$$\Rightarrow T \sin \theta = ma^2(L + x) - kx$$  \hspace{1cm} \cdots (ii)

**Step IV** Find the elongation in the spring from Eqs. (i) and (ii), we get

$$\tan \theta = \frac{ma^2(L + x) - kx}{mg}$$

$$\Rightarrow mg \tan \theta = ma^2(L + x) - kx$$

$$\Rightarrow x = \frac{mg \tan \theta - ma^2 L}{ma^2 - k}$$

$$\Rightarrow$$ Elongation in the spring,

$$x = \frac{g \tan \theta - \omega^2 L}{\omega^2 - k / m}$$

**(ii) Turning of a Vehicle Over a Level Road**

Following is the free body diagram for the car,

For car, \[ \Sigma F_x = f = \frac{mv^2}{R} \] \hspace{1cm} \cdots (i)

\[ \Sigma F_y = 0 \]

\[ \Rightarrow N = mg \] \hspace{1cm} \cdots (ii)

Now, maximum possible (or limiting) friction available is

$$f_{\text{max}} = \mu N = \mu mg$$

And writing $v_{\text{max}}$ for $v$ in Eq. (i), we have

$$t_{\text{max}} = \frac{mv_{\text{max}}^2}{R} = \mu mg$$

\[ \Rightarrow \quad v_{\text{max}} = \sqrt{\frac{\mu Rg}{R}} \]

Car easily turns without slipping, if friction is less than $t_{\text{max}}$ or speed of car is less than $v_{\text{max}}$. For speed faster than $v_{\text{max}}$, car skid over the turn.

**(iii) Turning of a Vehicle Over a Banked Road**

We resolve friction $f$ and normal reaction $N$ to get

$$N \cos \theta = mg + f \sin \theta$$  \hspace{1cm} \cdots (i)

$$N \sin \theta + f \cos \theta = \frac{mv^2}{R}$$  \hspace{1cm} \cdots (ii)

Maximum available friction is

$$f_{\text{max}} = \mu N$$  \hspace{1cm} \cdots (iii)

and from Eqs. (i), (ii) and (iii), we have

$$N = \frac{mg}{\cos \theta - \mu \sin \theta}$$

and

$$v_{\text{max}} = \sqrt{Rg \left( \frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)}$$

Comparing this speed with that of level road, we see that it is possible to take turn over a banked road with more speed.

- When $\mu = 0$, $v_{\text{max}} = \sqrt{Rg \tan \theta}$ at this speed you can take turn without any friction.
- To park car over incline, $\tan \theta \leq \mu_s$. 

**PHYSICS SPECTRUM < AUGUST 2016 < 47**
Example 6

TO FIND ELEVATION OF A RAILWAY TRACK

The radius of curvature of a railway track at a place, where the train is moving at a speed of 72 km/h is 625 m. The distance between the rails is 1.5 m. Find the angle and the elevation of the outer rail. So that there may be no side pressure on the rails. [Take, \( g = 9.8 \text{ m/s}^2 \)]

(a) \( \tan^{-1}(0.0653) = 3.78 \text{ cm} \)
(b) \( \tan^{-1}(12.3) = 7.78 \text{ cm} \)
(c) \( \tan^{-1}(0.05) = 7.45 \text{ cm} \)
(d) \( \cos^{-1}(4.25) = 5.66 \text{ cm} \)

**Sol.** (a) **Step I**
Given, \( v = 72 \text{ km/h} = \frac{72 \times 1000}{60 \times 60} = 20 \text{ m/s} \)
\( r = 625 \text{ m}, \quad b = 1.5 \text{ m}, \quad \theta = ? \), \( h = ? \)

**Step II** Angle of a outer rail track

\[
\tan \theta = \frac{v^2}{rg} = \frac{20 \times 20}{625 \times 9.8} = 0.065
\]
\( \tan \theta = 0.0653 = \theta = \tan^{-1}(0.0653) \)

**Step III** For elevation of the outer rail track, we have

\[
\tan \theta = \frac{h}{\sqrt{b^2 - h^2}} \Rightarrow h = \tan \theta \sqrt{b^2 - h^2}
\]
\( \Rightarrow \quad h^2 = (0.0653)^2(b^2 - h^2) \)
\( \Rightarrow \quad 1.004265h^2 = 0.0004265(1.5)^2 \)
\( h = 0.09775 \text{ m} = 9.78 \text{ cm} \)

Example 7

TO CALCULATE MAXIMUM SPEED OF A CAR

WITHOUT SKIDDING ON A BANKED ROAD

Car is moving along a banked road laid out as a circle or radius. What should be the banking angle \( \theta \), so that the car travelling at speed \( v \) needs no frictional force from the tyres to negotiate the turn? Also if the coefficient of friction between tyres and road is \( \mu_s \), then with what maximum speed can car enter the curve without sliding towards the top edge of the banked turn?

\[
(a) \quad \tan^{-1} \left( \frac{v^2}{rg} \right) \quad (b) \quad \tan^{-1} \left( \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \right) \quad (c) \quad \tan^{-1} \left( \frac{\tan \theta}{\sin \theta} \right) \quad (d) \quad \tan^{-1} \left( \frac{\sin \theta}{\cos \theta} \right)
\]

**Sol.** (b) **Step I**
Resolving \( x \) and \( y \)-components of normal reaction we get

\( N_x = N \sin \theta, \quad N_y = N \cos \theta \)

In our coordinate system, we apply Newton’s law in component form, we have

\[
\sum F_x = ma_x \quad \text{and} \quad \sum F_y = ma_y
\]
\( N \sin \theta = ma_x \quad \text{and} \quad N \cos \theta - w = ma_y \)
\( \Rightarrow \quad N \sin \theta = ma_x \quad \text{and} \quad N \cos \theta - w = ma_y \)

**Step II**
Since, the car travels along a circle, \( a_y = 0 \), \( a_x = \frac{v^2}{r} \)

\[
N \sin \theta = \frac{mv^2}{r} \quad \text{and} \quad N \cos \theta - w = ma_y
\]
\( \Rightarrow \quad \tan \theta = \frac{v^2}{rg} \quad \Rightarrow \quad \theta = \tan^{-1} \left( \frac{v^2}{rg} \right) \)

**Note** In formulating the above equations we have assumed friction to be absent (i.e. friction does not play any role in negotiating the turn).

**Step III**
If the driver moves faster than the speed mentioned above, friction force must act parallel to the road inwards, towards centre of the turn.

\[
(F)_x = F \cos \theta, \quad (F)_y = F \sin \theta
\]

Now,
\( \sum F_x = ma_x \quad \text{and} \quad \sum F_y = ma_y \)
\( N \sin \theta + F \cos \theta = \frac{mv^2}{r} \)
\( N \cos \theta - F \sin \theta - w = 0 \)

**Step IV**
As, the tyre rolls forwards slipping, the tendency of slipping arises sideways. Hence, we get

\( F = F_{\text{max}} = \mu_s \cdot N \)
\( \Rightarrow \quad N \sin \theta + \mu_s \cdot N \cos \theta = \frac{mv^2}{r} \)
\( N \cos \theta - \mu_s \cdot N \sin \theta = mg \)

On eliminating \( N \), we get
\( \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} = \frac{v^2}{rg} \)

Maximum speed of a car enter a curve without sliding towards the top edge of banked turn is given as
\( \Rightarrow \quad v = \sqrt{\frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta} \cdot rg} \)
(iv) **Overtaking of a Vehicle**

If a car moves in a circular path with speed more than the maximum speed, then it overturns and its inner wheel leaves the ground first.

Weight of the car is \( mg \), speed of the car is \( v \) and radius of circular path is \( r \). Separation between the centre of wheels of the car = \( 2x \)

Distance of the centre of gravity (G) of the car from the road level = \( y \)

So, that reaction on the inner wheel and outer wheel of the car by the ground is \( R_a \) and \( R_b \).

If a car moves in a circular path friction force \( F \) along the horizontal provides the required centripetal force i.e.

\[
F = \frac{mv^2}{r} \quad \text{...(i)}
\]

The condition for rotational equilibrium can be satisfied by taking the moment of force \( R_a, R_b \) and \( F \) about centre of gravity (G)

\[
Fy + R_a x = R_b x \quad \text{...(ii)}
\]

Since, there is no vertical motion, therefore

\[
R_a + R_b = mg \quad \text{...(iii)}
\]

After solving Eqs. (i), (ii) and (iii), we have

\[
R_a = \frac{1}{2} m \left[ g - \frac{v^2 y}{rx} \right] \quad \text{and} \quad R_b = \frac{1}{2} m \left[ g + \frac{v^2 y}{rx} \right] \quad \text{...(iv)}
\]

As, it can be said that from Eq. (iv),

if speed \( v \) increases, then value of reaction of the normal \( (R_a) \) on the inner wheel decreases.

\[
R_a = 0 \quad \text{i.e.} \quad \frac{v^2 y}{rx} = g
\]

or

\[
v = \sqrt{\frac{grx}{y}}
\]

i.e. the maximum speed of a car without overturning on a flat road is given by \( v = \sqrt{\frac{grx}{y}} \)

**Example 8**

**TO EVALUATE LIMITING SPEED OF A RACING CAR ON A CURVE BEYOND WHICH IT WILL OVERTURN**

A car of mass \( m \) travelling at speed \( v \) moves on a horizontal track, so that the centre of mass describes a circle of radius \( r \). What is the limiting speed beyond which the car will overturn, where \( 2a \) is the separation of inner and outer wheels and \( h \) is the height of the centre of mass above the ground?

\[
(a) \quad \frac{grx}{h} \quad \text{and} \quad (b) \quad \frac{3grx}{h}^2 \quad \text{and} \quad (c) \quad \frac{grx}{h} \quad \text{and} \quad (d) \quad \frac{grx}{h}^2
\]

**Sol. Step I** A car of mass \( m \) travelling at speed \( v \) moves on a horizontal track having radius \( r \) as shown in figure.

Sum of forces in vertical direction,

\[
R_1 + R_2 = mg \quad \text{...(i)}
\]

Sum of forces in horizontal direction,

\[
F_1 + F_2 = \frac{mv^2}{r} \quad \text{...(ii)}
\]

**Step II** Taking moment about the centre of mass G, we have

\[
(F_1 + F_2) h + R_1 a = R_2 a \Rightarrow F_1 + F_2 = (R_2 - R_1) \frac{a}{h} \quad \text{...(iii)}
\]

Combining this with Eq. (ii) to eliminate \( F_1 + F_2 \) gives

\[
R_2 - R_1 = \frac{hmv^2}{ar} \quad \text{...(iv)}
\]

**Step III** Now, we have two simultaneous Eqs. (i) and (v) for \( R_1 \) and \( R_2 \).

Solving these equations by adding and subtracting, we find that

\[
2R_1 = mg - \frac{hmv^2}{ar} \quad \text{and} \quad 2R_2 = mg + \frac{hmv^2}{ar} \quad \text{...(v)}
\]

**Step V** From Eq. (v), it is clear that inner wheels will leave the ground when \( R_1 \) will become zero and the car begins to overturn.

\[
mg = \frac{hmv^2}{ar}
\]

So, the limiting speed is given by

\[
v^2 = \frac{grx}{h}
\]

i.e.

\[
v = \sqrt{\frac{grx}{h}}
\]
(v) Bending of a Cyclist

When the cyclist goes round a curved path, a centripetal force is required. The force of friction between tyres and the road is too small to provide necessary centripetal force. That is a reason why a cyclist going round a curve leans inwards because the horizontal component of the normal reaction provides the necessary centripetal force.

![Diagram of a cyclist leaning in a curve]

Reaction, $R$ can be resolved into two rectangular components:

(i) $R \cos \theta$, along the vertical upward direction and

(ii) $R \sin \theta$, along the horizontal towards the centre of the circular track.

Now, $R \cos \theta = mg$ and $R \sin \theta = \frac{mv^2}{r}$

$\Rightarrow \tan \theta = \frac{v^2}{rg}$

Therefore, the cyclist should bend himself through an angle $\theta$ from the vertical, such that $\tan \theta = \frac{v^2}{rg}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

**Example**

**PROBLEM BASED ON BENDING OF CYCLIST**

A cyclist speeding at 18 km$h^{-1}$ on a level road takes a sharp circular turn of radius 3 m without reducing the speed and without bending towards the centre of the circular path. The coefficient of static friction between the tyres and the road is 0.1, moving on the road the cyclist will

(a) turn his cycle safely without slipping

(b) slip down

(c) not be able to turn

(d) None of the above

**Sol. Step I**

Since, a cyclist speeding at 18 km$h^{-1}$ on a level road takes a sharp circular turn of radius 3 m. It means that on a level road, force of friction alone is responsible for providing centripetal force for going along the circular turn.

**Step II**

Now, considering the angle through which cyclist bends, i.e.

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

or $\tan \theta = \mu = \frac{v^2}{rg}$

**Step III**

So, the maximum safe speed is given by

$v_{\text{max}} = \sqrt{\mu g}$

$\Rightarrow v_{\text{max}} = \sqrt{0.1 \times 3 \times 9.8} = \sqrt{2.94}$

$= 1.715\text{ms}^{-1}$

**Step IV**

Now, comparing the calculated maximum safe speed with the given actual speed.

Actual speed, $v = 18\text{kmh}^{-1} = 5\text{ms}^{-1}$

As the actual speed is greater than the maximum safe speed, so the cyclist will slip while taking the turn.

---

**Smart Practice**

**Single Option Correct Type**

1. The slope of the smooth banked horizontal road is $p$. If the radius of curve be $r$, then the maximum velocity with which a car negotiate the curve is given by

(a) $prg$  (b) $\sqrt{prg}$  (c) $\frac{p}{rg}$  (d) $\frac{p}{\sqrt{rg}}$

2. A particle describes a horizontal circle on the smooth inner surface of a conical funnel as shown in figure. If the height of the plane of the circle above the vertex is 9.8 cm. Find the speed of the particle.

(a) 294 m$s^{-1}$  (b) 0.98 m$s^{-1}$  (c) 1.42 m$s^{-1}$  (d) 4.32 m$s^{-1}$

3. A ball of mass 0.2 kg suspended by a string 60 cm long. Keeping the string always taut, the ball describes a horizontal circle of radius 30 cm. Angular speed of the ball is

(a) 8.14 rad s$^{-1}$  (b) 6.14 rad s$^{-1}$  (c) 4.34 rad s$^{-1}$  (d) 2 rad s$^{-1}$

4. A cyclist with combined mass of 80 kg going around a curved road with a uniform speed of 20 m$s^{-1}$. He has to bend inward by an angle $\theta = \tan^{-1}(0.50)$. With the vertical, then the force of friction between road surface and tyres will be

(a) 300 N  (b) 400 N  (c) 800 N  (d) 250 N
5. A bob of 0.25 kg is attached with a string of length 0.5 m. The bob is rotated on a horizontal circle. The maximum tension that string can bear is 225 N. Maximum possible value of angular velocity of bob (in rad/s) is (a) 9 (b) 18 (c) 30 (d) 60

6. Two identical discs are rotating about their axes in opposite directions.

Both discs have same radii R. At t = 0, points P and Q are facing each other. Correct graph of relative speed v_r between P and Q and time t is (T = time period of rotation) [JEE 2012]

(a) ![Graph (a)](image1)
(b) ![Graph (b)](image2)
(c) ![Graph (c)](image3)
(d) ![Graph (d)](image4)

7. A simple pendulum is oscillating without damping. Displacement of bob is less than the maximum. Its acceleration vector is correctly shown in (a) ![Acceleration vector (a)](image5) (b) ![Acceleration vector (b)](image6) (c) ![Acceleration vector (c)](image7) (d) ![Acceleration vector (d)](image8) [JEE Advanced 2011]

8. A particle P will be rotating at a height of 0.2 m inside a hemispherical bowl of 0.5 m radius.

Angular speed of bowl must be

(a) \( \frac{10}{\sqrt{3}} \) rad/s
(b) \( 10\sqrt{3} \) rad/s
(c) 10 rad/s
(d) \( \sqrt{20} \) rad/s [JEE Advanced 2007]

9. Two particles each of mass 'm' are attached to two ends of a light string of length 'L', which passes through a hole at the centre of a smooth table.

One of particle rotates over the table with angular velocity \( \omega_1 \), in radius \( l_1 \) and other particle describes a conical pendulum with angular speed \( \omega_2 \). If \( l_2 \) is length of string below the table, then correct option is (a) \( \frac{l_1}{l_2} = \frac{\omega_2}{\omega_1} \) (b) \( \frac{l_1}{l_2} = \left( \frac{\omega_2}{\omega_1} \right)^2 \) (c) \( \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{m \cdot L}{g} \) (d) \( \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{l_1 \sin \theta}{g} \) [JEE Advanced 2002]

Integer Type

10. A string of length L is fixed at one end and connected to a bob of mass m at the other end. The string makes \( \omega/\pi \) revolutions per second around the vertical axis through the fixed end. If tension in the string is 64 mL, then \( \omega \) is

11. A particle describes a horizontal circle on the smooth inner surface of a conical funnel as shown in figure. If the height of the plane of the circle above the vertex is 9.8 cm. So, the speed of the particle is \( 0.98 \times 10^3 \) ms\(^{-1} \). The value of \( x + 1 \) is

More Than One Answer Correct Type

12. For a particle performing uniform circular motion, choose the correct statement(s) from the following
(a) magnitude of particle velocity remains constant
(b) particle velocity remains directed perpendicular to radius vector
(c) Direction of acceleration keeps changing as particle moves
(d) Angular momentum is constant in magnitude but direction keeps changing

13. A ball of mass $m$ is suspended from a ceiling through a string of length $L$. If the ball moves in horizontal circle of radius $a$, then
(a) tension in the string is $\frac{mgl}{(L^2 - a^2)^{3/2}}$
(b) tension in the string is $\frac{2mgl}{(L^2 + a^2)^{3/2}}$
(c) speed of the ball is $\frac{\sqrt{ga}}{(L^2 - a^2)^{3/4}}$
(d) speed of the ball is $\frac{2a\sqrt{g}}{(L^2 + a^2)^{1/4}}$

Comprehension Type

When a vehicle rounds a curve, it requires some centripetal force $\frac{mv^2}{r}$. If the road is unbanked, the necessary force is provided by the force of friction between the tyres and road. To avoid skidding, the speed of vehicle must be $\leq \sqrt{\frac{\mu grx}{h}}$, where $2x$ is the wheel base and $h$ is height of centre of gravity, above the road. The dependence on friction can be avoided, if the road is suitably banked. The safe speed, then rises to $\sqrt{\frac{gr}{h}} \tan \theta$. In no case, the speed limits depends upon mass of the vehicle.

Read the above passage and answer the following questions, when mass of car is 800 kg, wheel base is 1.1 m, height of centre of gravity is 50 cm, banking angle is 30° and radius of curve is 200 m. (Take, $g = 9.8 \text{ ms}^{-2}$ and $\mu = 0.2$)

14. The safe speed to avoid skidding on the unbanked curve is
(a) 9.8 ms$^{-1}$
(b) 19.8 ms$^{-1}$
(c) 10 ms$^{-1}$
(d) 1.98 ms$^{-1}$

15. The safe speed to avoid toppling on unbanked curve is
(a) 19.8 ms$^{-1}$
(b) 198 ms$^{-1}$
(c) 46.4 ms$^{-1}$
(d) 1.98 ms$^{-1}$

16. If the speed of car is more than this safe speed, then car would topple
(a) inwards
(b) outwards
(c) sometimes inwards
(d) cannot say

17. What time $t$ is required for block A to move 20 m?
(a) 4.47 s
(b) 3.47 s
(c) 5.47 s
(d) 6.47 s

18. The velocity of A at time $t$ (mentioned in the above question) (in ms$^{-1}$) is
(a) 9.94
(b) 8.94
(c) 7.94
(d) 6.94

19. The velocity of B at time $t$ (mentioned in the above question) (in ms$^{-1}$) is
(a) 4.7
(b) 3.7
(c) 5.7
(d) 6.7

Matching Type

20. A gyroscopic is spinning at angular velocity $\omega_0$. The power is shut off and the gyroscope begins to slow down with its angular velocity described by $\omega = \omega_0 e^{-t/T}$, where $t$ is the time and $T$ is a positive constant. Now, match the following columns and select the correct option from the codes given below:

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. The time in which the gyroscope reaching half of its initial angular velocity.</td>
<td>p. $\frac{\omega_0 T}{4\pi}$</td>
</tr>
<tr>
<td>ii. Initial angular acceleration of the gyroscope</td>
<td>q. $-\frac{\omega_0}{2 T \ln 2}$</td>
</tr>
<tr>
<td>iii. The average angular acceleration between the time, the power is cut-off and the instant it reaches half speed</td>
<td>r. $-\frac{\omega_0}{T}$</td>
</tr>
<tr>
<td>iv. Number of revolutions the gyroscope turns between the time the power is cut-off and the instant it reaches half speed</td>
<td>s. $T \ln 2$</td>
</tr>
</tbody>
</table>

Codes

<table>
<thead>
<tr>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>i</th>
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<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>r</td>
<td>q</td>
<td>p</td>
<td>(b)</td>
<td>p</td>
<td>q</td>
<td>r</td>
</tr>
<tr>
<td>(c)</td>
<td>q</td>
<td>r</td>
<td>s</td>
<td>(d)</td>
<td>p</td>
<td>r</td>
<td>q</td>
</tr>
</tbody>
</table>

Answers

1. (b)
2. (b)
3. (c)
4. (b)
5. (d)
6. (a)
7. (c)
8. (a)
9. (b)
10. (4)
11. (1)
12. (a,b,c,d)
13. (a,c)
14. (b)
15. (c)
16. (b)
17. (a)
18. (b)
19. (d)
20. (a)
Cylindrical Surface

**Problem No. 3.23**

An infinitely long cylindrical surface of a circular cross-section is uniformly charged lengthwise with the surface density \( \sigma = \sigma_0 \cos \phi \), where \( \phi \) is the polar angle of the cylindrical coordinates system whose \( Z \)-axis coincides with the axis of the given surface. Find the magnitude and direction of the electric field strength vector on the \( Z \)-axis.

**Physiks Funda**

Since, surface charge density is given, i.e. \( \sigma = \sigma_0 \cos \phi \), where, \( \phi \) is the polar angle of cylindrical coordinates system.

**Sol.**

**Step I** The given cylinder behaves as a long bamboo. So the strip of the cylinder along length of the cylinder as long thread. The top view of cylinder is like a circular ring as shown in figure.

**Step II** We consider a long strip of width \( dt = Rd\phi \) as shown in figure.

If length of cylinder is \( l \) (\( l \rightarrow \infty \)). The area of strip is \( dA = ld\phi = lRd\phi \). The charge on the strip is \( dq = \sigma dA = \sigma_0 \cos \phi lRd\phi = \sigma_0 l/2 \cos \phi d\phi \)

**Step III** The linear charge density of considered strip is

\[
\lambda = \frac{dq}{l} = \frac{\sigma_0 lR \cos \phi d\phi}{l} \cos \phi = \frac{\sigma_0}{2} \cos \phi
\]

The electric field at point \( O \) due to considered long strip is

\[
dE = \frac{\lambda}{2\pi \varepsilon_0 R} = \frac{\sigma_0}{2\pi \varepsilon_0} \cos \phi d\phi
\]

\[
dE = -dE \cos \phi \hat{i} - dE \sin \phi \hat{j}
\]

**Step IV** The component of electric field along \( X \)-axis due to whole cylinder is

\[
E_x = \int_{\phi=0}^{\phi=2\pi} -dE \cos \phi = -\frac{\sigma_0}{2\pi \varepsilon_0} \int_0^{2\pi} \cos^2 \phi d\phi = \frac{-\sigma_0}{2\pi \varepsilon_0} \int_0^{2\pi} \left(1 + \cos 2\phi \right) d\phi = \frac{-\sigma_0}{2\pi \varepsilon_0} \int_0^{2\pi} \left(1 + \cos 2\phi \right) d\phi = \frac{-\sigma_0}{2\pi \varepsilon_0}
\]

**Step V** The component of electric field due to whole cylinder along \( Y \)-axis is

\[
E_y = \int_{\phi=0}^{\phi=2\pi} -dE \sin \phi = -\frac{\sigma_0}{2\pi \varepsilon_0} \int_0^{2\pi} \cos \phi \sin \phi d\phi = \frac{-\sigma_0}{4\pi \varepsilon_0} \int_0^{2\pi} \sin 2\phi d\phi = 0
\]

\(
E = E_x \hat{i} + E_y \hat{j}
\)

i.e.

\[
E = \frac{\sigma_0}{2\pi \varepsilon_0} \left( \text{along the direction } \phi = \pi \right)
\]
Spherical Charge Distribution

Problem No. 3.27

A space is filled up with a charge with volume density \( p = \rho_0 e^{-\alpha r^3} \), where \( \rho_0 \) and \( \alpha \) are positive constants, \( r \) is the distance from the centre of this system. Find the magnitude of the electric field strength vector as a function of \( r \). Investigate the obtained expression for the small and large values of \( r \), i.e. at \( \alpha r^3 << 1 \) and \( \alpha r^3 >> 1 \).

**PHYSIKS FUNDA**

The electric field on the surface of spherical distribution of charge is \( E = \frac{q}{4\pi\varepsilon_0 r^2} \).

This result is applicable for uniform as well as non-uniform distribution of charge. Thus, it does not depend upon charge outside the surface.

**Sol.** Step I Draw a sphere of radius \( r \) as shown in figure below.

![Diagram](image1)

Step II Calculate the net charge enclosed by the sphere as shown in figure.

For this, we consider a hollow concentric sphere of radius \( r \) and thickness \( dr \). The volume of considered element is \( dV = 4\pi r^2 dr \).

The electric charge in the considered element is

\[ dq = pdV = \rho_0 e^{-\alpha r^3} \cdot 4\pi r^2 dr \]

\[ dq = 4\pi \rho_0 r^2 e^{-\alpha r^3} dr \]

\[ = \frac{2\pi \rho_0}{3\varepsilon_0 \alpha^3} (1 - e^{-\alpha r^3}) \]

Step III Apply the concept of the problem, electric field on the surface of spherical distribution of charge,

\[ E = \frac{\rho_0}{3\varepsilon_0 \alpha^3} (1 - e^{-\alpha r^3}) \]

[radially outwards]

Thin Circular Ring

Problem No. 3.30

There are two thin wire rings, each of radius \( R \), whose axes coincide. The charges of the rings are \( +q \) and \( -q \). Find the potential difference between the centres of the rings separated by a distance \( a \).

**PHYSIKS FUNDA**

Electric potential at point \( P \) (on the axis) due to ring is

\[ \phi = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 \sqrt{R^2 + a^2}} \]

**Sol.** Step I Determine the net potential at the centre of first ring. The electric potential on the centre of first ring is

\[ \phi_1 = \phi_1' + \phi_1'' \]

where, \( \phi_1' \) = Electric potential due to first ring on its centre

\[ = \frac{q}{4\pi\varepsilon_0 R} \]

and \( \phi_1'' \) = Electric potential due to second ring at the centre of first ring

\[ = \frac{-q}{4\pi\varepsilon_0 \sqrt{R^2 + a^2}} \]

Step II Determine the net potential at the centre of second ring,

\[ \phi_2 = \phi_2' + \phi_2'' \]

where, \( \phi_2' \) = Electric potential due to second ring as its centre

\[ = \frac{-q}{4\pi\varepsilon_0 R} \]

and \( \phi_2'' \) = Electric potential due to first ring on the centre of second ring

\[ = \frac{q}{4\pi\varepsilon_0 \sqrt{R^2 + a^2}} \]
Step III Determine potential difference.

\[ \Delta \phi = \phi_1 - \phi_2 \]

On putting the values of \( \phi_1 \) and \( \phi_2 \), we get

\[ \Delta \phi = \frac{q}{2 \pi \varepsilon_0 R} \left[ 1 - \frac{1}{\sqrt{1 + \frac{a^2}{R^2}}} \right] \]

### Hemispherical Surface

#### Problem No. 3.32

Find the electric field potential and strength at the centre of a hemisphere of radius \( R \) charged uniformly with the surface density \( \sigma \).

**PHYSIKS FUNDA**

The hemisphere can be divided into the number of ring elements. The electric field at the point \( O \) is vector sum of the field due to each such rings.

**Sol.**

**Step I** Let us consider a ring element located at an angle \( \theta \) with the reference line \( ON \) as shown in figure and subtending an angle \( d\theta \) at the centre \( O \). The area of considered element is

\[ dA = 2\pi R^2 \sin \theta \ d\theta \]

\[ OC = x = R \cos \theta \]

**Step II** The charge on considered element is

\[ dq = \sigma 2\pi R^2 \sin \theta \ d\theta \]

\[ dE = \frac{x dq}{4\pi \varepsilon_0 (R^2 + x^2)^{3/2}} \]

\[ dE = \frac{2\pi \sigma \sin \theta \cos \theta \ d\theta}{4\pi \varepsilon_0} = \frac{\sigma \sin 2\theta \ d\theta}{4\pi \varepsilon_0} \]

\[ E = \int dE = \frac{\sigma}{4\pi \varepsilon_0} \]

Along \( NO \)

**Step III** Similarly, electric potential due to considered ring element at point \( O \) is

\[ d\phi = \frac{dq}{4\pi \varepsilon_0 R} = \frac{\sigma R \sin \theta \ d\theta}{2 \varepsilon_0} \]

\[ \phi = \int d\phi = \frac{\sigma R}{2 \varepsilon_0} \int_0^{\pi/2} \sin \theta \ d\theta \]

\[ \phi = \frac{\sigma R}{2 \varepsilon_0} \]

### Worked Out Problem

#### Problem No. 3.46

**Problem: Electric Dipole Moment along a Long Thread**

A dipole with an electric moment \( P \) is located at a distance \( r \) from a long thread charged uniformly with a linear density \( \lambda \). Find the force \( F \) acting on the dipole, if the vector \( P \) is oriented

(a) along the thread

(b) along the radius vector \( r \)

(c) at right angles to the thread and the radius vector \( r \).

**PHYSIKS FUNDA**

Electric field due to a long thread is

\[ E = \frac{\lambda}{2\pi \varepsilon_0 r} \]

along increasing \( r \).

**Sol.**

**Step I (a)** The arrangement is shown in figure, the net force on the dipole is \( F = qE - qE = 0 \)

**Step II (b)** The electric field at the site of charge \( (-q) \) is

\[ E_1 = \frac{\lambda}{2\pi \varepsilon_0 (r - a)} \]

As shown in figure, the net force on the dipoles is

\[ F = qE_1 - qE_2 = q (E_1 - E_2) \]

\[ \frac{\lambda}{2\pi \varepsilon_0 (r - a)} - \frac{\lambda}{2\pi \varepsilon_0 (r + a)} \]

\[ = \frac{q \lambda}{2\pi \varepsilon_0} \left( \frac{1}{r - a} - \frac{1}{r + a} \right) \]

\[ = \frac{q \lambda}{2\pi \varepsilon_0} \left( \frac{2a}{(r^2 - a^2)} \right) \]

\[ = \frac{(q \times 2a) \lambda}{2\pi \varepsilon_0 (r^2 - a^2)} \]

But \( r >> a \), so \( a^2 \) can be neglected with respect to \( r^2 \).

\[ \therefore F = \frac{\mu \lambda}{2\pi \varepsilon_0 r^2} \]

Since, resultant force on the electric dipole is in the opposite of dipole moment, therefore

\[ F = \frac{-\mu \lambda}{2\pi \varepsilon_0 r^2} \]  

[In vector form]
Step III (c) In the figure, the long thread is placed perpendicular to the plane of paper.

Here, \[ E_1 = \frac{\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \]
and \[ E_2 = \frac{\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \]
Also, \[ F_1 = qE_1 = \frac{q\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \]
and \[ F_2 = qE_2 = \frac{q\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \]
\[ \therefore F_1 = F_2 = F_0 = \frac{q\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \] (i)

Step IV In force diagram as shown in figure, we have

\[ F = 2F_0\cos\theta \]

On putting the values of \( F_0 \) and \( \cos\theta \) in Eq. (i), we get

\[ F = \frac{2q\lambda}{2\pi\varepsilon_0(\sqrt{r^2 + a^2})} \times \frac{a}{\sqrt{r^2 + a^2}} = \frac{\lambda (q \times 2a)}{2\pi\varepsilon_0(r^2 + a^2)} = \frac{p\lambda}{2\pi\varepsilon_0(r^2 + a^2)} \]

But \( r >> a, a^2 \) may be neglected with respect to \( r^2 \), we get

\[ \therefore \text{Net force, } F = \frac{p\lambda}{2\pi\varepsilon_0 r^2} \]

(along the direction of dipole moment)
Chapterwise Collection of Best Assertion-Reason Problems for Medical Entrances

Directions (Q. Nos. 1-17) These questions consist of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following five responses.
(a) If both Assertion and Reason are true and the Reason is the correct explanation of the Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of the Assertion.
(c) If Assertion is true but Reason is false.
(d) If the Assertion and Reason both are false.
(e) If Assertion is false but Reason is true.

1. Assertion The centre of mass of an isolated system has a constant velocity.
   Reason If centre of mass of an isolated system is already at rest, it remains at rest.

2. Assertion A particle is moving on a straight line with a uniform velocity, its angular momentum is always zero.
   Reason The momentum is zero particle moves with a uniform velocity. [AIIMS 2010]

3. Assertion The centre of mass of an electron and photon, when released moves faster towards proton.
   Reason Proton is heavier than electron.

4. Assertion It is harder to open and shut the door if we apply force near the hinge.
   Reason Torque is maximum at hinge of the door.

5. Assertion Radius of gyration of body is a constant quantity.
   Reason The radius of gyration of a body about an axis of rotation may be defined as the root mean square distance of the particle from the axis of rotation.

6. Assertion The total kinetic energy of a rolling solid sphere is the sum of translational and rotational kinetic energies.
   Reason For all solid bodies, total kinetic energy is always twice the translational kinetic energy. [AIIMS 2013]

7. Assertion Torque is equal to rate of change of angular momentum.
   Reason Angular momentum is conserved.

8. Assertion The centre of mass of a body will change with the change in shape and size of the body.
   Reason \[ r = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} \]

9. Assertion The velocity of a body at the bottom of an inclined plane given height is more when it slides down the plane compared to when it rolls down the same plane.
   Reason In rolling down, a body acquires both, kinetic energy of translation and kinetic energy of rotation.

10. Assertion A ladder is more opt to slip when you are high on it than when you just begin to climb.
    Reason At the high on ladder, the torque is large and on climbing up the torque is small. [AIIMS 2014]

11. Assertion When ice on polar caps of earth melts, duration of the day increases.
    Reason \[ L = 10T = \frac{2\pi I}{T} = \text{constant} \]

12. Assertion Moment of inertia of circular ring about a given axis is more than moment of inertia of the circular disc of same mass and same size, about the same axis.
    Reason The circular ring is hollow so its moment of inertia is more than circular disc which is solid.

13. Assertion At the centre of earth, a body has centre of mass but no centre of gravity.
    Reason This is because \( g = 0 \) at the centre of earth.

14. Assertion There are two propellers in a helicopter.
    Reason Angular momentum is conserved. [AIIMS 2015]

15. Assertion Inertia and moment of inertia are same quantities.
    Reason Inertia represents the capacity of a body to oppose its states of motion or rest.

16. Assertion A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling motion).
    Reason For perfectly rolling motion, work done against friction is zero.

17. Assertion If earth shrink (without change in mass) to half its present size, length of the day would become 6 hours.
    Reason As size of the earth change its moment of inertia changes.
18. **Assertion** Due to global warming duration of the day of earth is increasing.

   **Reason** Angular momentum of earth remains constant.

19. **Assertion** Angular momentum of earth remains constant even when mean temperature of earth increases.

   **Reason** There is no net force on earth.

20. **Assertion** Earth rotates round the sun in an elliptical orbit.

   **Reason** Centre of mass of earth sun system lies near surface of sun.

---

**Answers with Explanation**

1. (b) External force on the system $F_{ext} = M \frac{d}{dt}(\mathbf{v}_{cm})$. If system is isolated i.e., $F_{ext} = 0$, then $\mathbf{v}_{cm}$ = constant. If initially the velocity of centre of mass is zero, then it will remain zero.

2. (d) When particle moves with constant velocity $\mathbf{v}$, then its linear momentum has some finite value, $p = mv$. Angular momentum 

   $(L) = \text{Linear momentum} \times \text{perpendicular distance of line of action of linear momentum from the point of rotation (d)}$.

   So, if $d \neq 0$, then $L \neq 0$, but if $d = 0$, then $L$ may be zero. So, we can conclude that angular momentum of a particle moving with constant velocity is not always zero.

3. (e) The position of centre of mass of electron and proton remains at rest. As their motion is due to internal force of electrostatic attraction, which is conservative force no external force is acting on the two particles, therefore centre of mass remain at rest.

4. (c) Torque $= \text{Force} \times \text{perpendicular distance of line of action of force from the axis of rotation (d)}$.

   Hence, for a given applied force, torque or true tendency of rotation will be high for large value of $d$. If distance $d$ is smaller, then greater force is required to case the same torque, hence it is harder to open or shut down the door by applying a force near the hinge.

5. (e) Radius of gyration of body is not a constant quantity. The value changes with the change in location of the axis of rotation. Radius of gyration of a body about a given axis is given is:

   $$ R = \sqrt{\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}} $$

6. (c) Given that total kinetic energy of a rolling solid sphere is the sum of translational and rotational kinetic energies.

   i.e.,

   $$ K_N = K_R + K_I $$

   This equation is correct for any body which is rolling without slipping.

   For the ring and hollow cylinder only $K_R = K_T$

   i.e.,

   $$ K_N = 2K_I $$

7. (b) As, torque $(t) = \frac{dL}{dt}$ and angular momentum $Z = I\omega$

   i.e. torque is equal to the rate of change of angular momentum.

   So, both assertion and reason are true but reason is not the correct explanation of the assertion.

8. (a) Position vector of centre of mass depends on masses of particles and their location. Therefore, change in shape size of body do change the centre of mass.

9. (e) In sliding down, the entire potential energy is converted only into linear kinetic energy. In rolling down, a part of same potential energy is converted into kinetic energy of rotation. Therefore, velocity acquired is less.

10. (a) When a person is high up on the ladder. Then a large torque is produced due to his weight about the point of contact between the ladder and the floor whereas when he starts climbing up the torque is small. Due to this reason the ladder is more opt to slip when one is high upon it.

11. (a) Both the assertion and reason are true and reason is a correct explanation of the statement. In fact, as ice on polar caps of earth melts, mass near the polar axis spreads out, $I$ increases. Therefore, $T$ increases i.e. duration of day increases.

12. (b) In the case of circular ring. The mass is concentrated on the rim (at maximum distance from the axis). Therefore, moment of inertia increase as compared to that in circular disc.

13. (a) At the centre of earth, $g = 0$. Therefore a body has no weight at the centre of earth and hence no centre of gravity. But centre of mass of a body has nothing to do with gravity. Therefore, centre of mass exists.

14. (b) Both the assertion and reason are true, but the reason is not correct explanation of the assertion. In fact, if helicopter had only one propeller, the helicopter itself will turn in opposite direction to conserve the angular momentum.

15. (e) There is a difference between inertia and moment of inertia of a body. The inertia of a body depends only upon the mass of the body but the moment of inertia of a body about an axis not only depends upon the mass of the body but also upon the distribution of mass about the axis of rotation.

16. (b) Rolling occurs only on account of friction which is a tangentially force capable of providing torque. When the inclined plane is perfectly smooth, body will simply slip under the effect of its own weight.

17. (a) When earth shrinks it angular momentum remains constant i.e.

   \[ L = I\omega = \frac{2}{5}mR^2 \times \frac{2\pi}{T} = \text{constant} \]

   \[ \therefore T \sim I \sim R^2 \]

   It means if size of the earth change, then its moment of inertia changes. In the problem, radius becomes half so time period (length of the day) will become 1/4 of the present value i.e., 24/4 = 6 h.

18. (a)

19. (c) Angular momentum of earth remains constant even when there is a force towards focus of its elliptical orbit because force is perpendicular to angular momentum and so it does not affects value of angular momentum. A is correct, R is incorrect.

20. (a)
Concept Equilibrium of a Body or System

Perception Generally, students think that for the equilibrium of a body or a system, the sum of forces acting on it (i.e., net force) is zero.

Explanation For the equilibrium of a body or a system when forces acting on it are concurrent, then
\[ \sum F_{\text{net}} = \text{vector sum of all the forces} = 0 \]
when forces acting on it are not concurrent and body or system can rotate about an axis, then
\[ \sum F_{\text{net}} = \text{vector sum of all the forces} = 0 \quad \text{and} \]
\[ \sum \tau = \text{vector sum of moment of all the forces} = 0 \]

Concept Non-Uniformly Accelerated Motion

Perception Most of the students believe that if the object moves at high speed, then its acceleration is also high and if the object moves with a low velocity, then acceleration is also law.

Explanation As, high velocity does not mean high acceleration. Acceleration can be high, low or zero in high velocities. It is only related to the change in velocity. If the change in velocity is high, then acceleration will be high, if the change in velocity is low, then the acceleration is also low.

Concept Measurement of Pressure

Perception Pressure is a vector quantity.

Explanation No, pressure is a scalar quantity because at any point inside the liquid, the pressure is exerted equally in all directions which shows that a definite direction is not associated with hydrostatic pressure.

Concept Conservation of Energy

Perception The mechanical energy of a system is always positive.

Explanation The mechanical energy of a system may be positive, negative or zero, depending on the relative magnitude of kinetic and potential energies.

As, kinetic energy (always positive) + potential energy (positive, zero or negative) = Total energy \( TE \) (positive, zero or negative).

Concept Determination of Direction of Force and Acceleration

Perception It is a general thing that acceleration of a body or system is always in the direction of force.

Explanation Acceleration produced in the body is always in the direction of net unbalanced force acting on the body. It should be differentiated in the mind force and net force (i.e., sum of all the forces acting on a body) are two different things.

![Diagram](image)

One of the component of force \( F \) (i.e., \( F \sin \theta \)) is countered by \( mg \) (weight of body). So, the motion of body in horizontal line is caused only by the horizontal component of force i.e., \( F \cos \theta \). In this case, if the acceleration of body is \( 'a' \) (in horizontal direction), then we can write, \( F \cos \theta = ma \).

Concept Acceleration due to Gravity and Gravitational Field Intensity

Perception It is a general misconception of student's that acceleration due to gravity and gravitational field intensity near the earth's surface are the same.

Explanation It should be clearly understood that the intensity of gravitational field and the acceleration due to gravity are not same. These are two separate physical quantities having equal magnitude and direction.

Out of these two physical quantities, one is force extend on unit mass while other is acceleration (ability to change the velocity of a body with respect to time whenever body is moved through the gravitational field at some angle from horizontal).

To be Continued at Page 80
LAWS OF MOTION
FOR CLASS XI

Topicwise Collection of best subjective problems

**Very Short Answer Type Questions [1 Mark]**

1. A heavy point mass tied to the end of string is whirled in a horizontal circle of radius 20 cm with a constant angular speed. What is the angular speed if the centripetal acceleration is 980 cm s\(^{-2}\)?

2. Carts with rubber tyres are easier to ply than those with the iron types. Explain.

3. A body is acted upon by a number of external forces. Can it remain at rest? \([\text{HOTS}]\)

4. A force of 36 dyne is inclined to the horizontal at an angle of 60°. Find the acceleration in a mass of 18 g that moves in a horizontal acceleration.

5. An impulse is applied to a moving object with a force at an angle of 20° w.r.t. velocity vector, what is the angle between the impulse vector and change in momentum vector? \([\text{HOTS}]\)

**Short Answer Type I Questions [2 Marks]**

6. A passenger of mass 72.2 kg is riding in an elevator while standing on a platform scale. What does the scale read when the elevator cab is

(i) descending with constant velocity?

(ii) ascending with constant acceleration 3.5 m/s\(^2\)?

7. A block of mass M is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is \(\mu\) and the acceleration due to gravity is \(g\). Calculate the minimum force required to be applied by finger to hold the block against the wall? \([\text{HOTS}]\)

8. If the speed of stone is increased beyond the maximum permissible value and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks.

(i) The stone moves radially outwards.

(ii) The stone flies off tangentially from the instant the string breaks.

(iii) The stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle.

9. A woman throws an object of mass 500 g with a speed of 25 m/s.

(i) What is the impulse imparted to the object?

(ii) If the object hits a wall and rebounds with half the original speed. What is the change in momentum of the object?

10. Why does a child feel more pain when he falls down on a hard cement floor than when he falls on the soft muddy ground in the garden?

**Short Answer Type II Questions [3 Marks]**

11. A stream of water flowing horizontally with a speed of 15 m/s gushes out of a tube of cross-sectional area \(10^{-2}\) m\(^2\) and hits a vertical wall nearby. What is the force exerted on the wall by the impact of the water, assuming it does not rebound?

12. A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is \(10^5\) kg. What provides the centripetal force required for this purpose? The engine or the rail? What is the angle of banking required to prevent wearing out the rail? \([\text{HOTS}]\)

13. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with speed 40 rev/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand tension of 200 N?
14. A stone of mass \( m \) tied to the end of a string revolves in a vertical circle of radius \( R \). The net force at the lowest and higher points of the circle directed vertically downwards are (choose the correct alternative).

<table>
<thead>
<tr>
<th>Lowest point</th>
<th>Highest point</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( T_1 - mg )</td>
<td>( mg + T_2 )</td>
</tr>
<tr>
<td>(ii) ( mg + T_1 )</td>
<td>( mg - T_2 )</td>
</tr>
<tr>
<td>(iii) ( mg + T_1 - (m v_1^2) / R )</td>
<td>( mg - T_2 + (m v_2^2) / R )</td>
</tr>
<tr>
<td>(iv) ( mg - T_1 - (m v_1^2) / R )</td>
<td>( mg + T_2 + (m v_2^2) / R )</td>
</tr>
</tbody>
</table>

Here, \( (T_1, T_2) \) and \( (v_1, v_2) \) denote the tensions in the string (and the speeds of the stone) at the lowest and highest points respectively.

15. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble.

(i) During its upward direction.
(ii) During its downward direction.
(iii) At the highest point where it is momentarily at rest.

Do your answer change if the pebble was thrown at an angle of 45° with the horizontal direction? Ignore air resistance.

Value Based Question [4 Marks]

16. In physics classroom teacher asks to his student that a car driver along with his car has to move on a level turn of radius 45 m. If the coefficient of static friction between the tyre and the road is \( \mu_s = 0.2 \).

Then, (i) what will be the speed of car so that the car does not skid?
(ii) what type of teaching skill of the teacher is being represented here?

Long Answer Type Questions [5 Marks]

17. Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg.

(i) Just after it is dropped from the window of a stationary train.

(ii) Just after it is dropped from the window of a train at a constant velocity of 36 km/h.

(iii) Just after it is dropped from the window of a train accelerating with 1 m/s².

(iv) Lying on the floor of a train which is accelerating with 1 m/s², the stone being at rest relative to the train. Neglect air resistance throughout.

18. Two bodies \( A \) and \( B \) of masses 5 kg and 10 kg in contact with each other rest on a table is against a rigid wall as shown in figure. The coefficient of friction between the bodies and table is 0.15. A force of 200 N is applied horizontally at body \( A \). What are

(i) the reaction of the wall?
(ii) the action-reaction forces between \( A \) and \( B \)?

Ignore differences between \( \mu_s \) and \( \mu_k \).

19. An object of mass 5 kg, attached to a spring scale, rests on a frictionless, horizontal surface as in given figure. The spring scale, attached to the front end of a boxcar, has a constant reading of 18 N when the car in motion.

(i) If the spring scale reads zero when the car is at rest, determine the acceleration of the car.
(ii) What constant reading will the spring scale show if the car moves with constant velocity?
(iii) Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.

20. A small block is at rest on the floor at the front of a rail-road boxcar that has length \( l \). The coefficient of kinetic friction between the floor of the car and the block is \( \mu_k \). The car, originally at rest, begins to move with acceleration \( a \). The block slides back horizontally until it hits the back wall of the car. At that moment, what is its speed

(i) relative to the car?
(ii) relative to earth?

Solutions

4. \( F = 36 \) dyne at an angle of 60°.

\[ \therefore \text{Component of force along x-direction:} \\
\quad F_x = F \cdot \cos 60° \]
\[ = 36 \times \frac{1}{2} = 18 \text{ dyne.} \]

But
\[ F = m \cdot a \]
\[ \Rightarrow a = \frac{F}{m} = \frac{18}{18} = 1 \text{ cm/s}^2 \]

5. Impulse and change in momentum are along the same direction. Therefore, the angle between the two vectors is zero degree.
6. Given, mass \( m = 72.2 \) kg
   Gravity acceleration \( g = 9.8 \) m/s\(^2\)
   Scale reading = apparent weight \( R = ? \)
   (i) While descending with constant velocity, \( a = 0 \).
   Therefore, \( F = mg \)
   \[ R = 72.2 \times 9.8 \Rightarrow R = 707.56 \text{ N} \]
   (ii) While ascending with constant acceleration
   \[ a = 32 \text{ m/s}^2 \]
   \[ R = m(g + a) \]
   \[ R = 72.2 \times (9.8 + 3.2) = 938.8 \text{ N} \]

7. Given, mass of the block = \( M \)
   Coefficient of friction between the wall and the block is \( \mu \).
   Let a force \( F \) be applied on the block to hold the block against
   the wall. The normal reaction of mass be \( N \) and the force of
   friction acting upward be \( f \).
   In equilibrium, vertical and horizontal forces should be balanced
   separately.

\[
F - N = mg \\
\therefore \quad f = \mu N \\
\text{and} \quad F = N \quad \text{(i)}
\]

But force of friction \( f = \mu N \)
\[
\therefore \quad \mu F = M \cdot g \Rightarrow F = \frac{Mg}{\mu} \quad \text{(iii)} \quad \text{[using Eq. (ii)]}
\]

8. The second part correctly describes the trajectory of the stone
   after the spring breaks because when a stone tied to one end of a
   string is whirled round in a circle when velocity of the stone at any
   point is along the tangent at that point. If the string breaks
   suddenly, then stone flies off tangentially, along the direction of its
   velocity.

9. Mass of the object \( m = 500 \) g = 0.5 kg
   Speed of the object \( v = 25 \) m/s
   (i) Impulse imparted to the object
   \[ = \text{change in the momentum} \]
   \[ = m \cdot v - m \cdot u = m(v - u) \]
   \[ = 0.5(25 - 0) = 12.5 \text{ N-s} \]
   (ii) Velocity of the object after rebounding,
   \[ = \frac{25}{2} \text{ m/s} \]
   \[ v' = -12.5 \text{ m/s} \]
   \[ = \text{change in momentum} = m(v' - v) \]
   \[ = 0.5(-12.5 - 25) = -18.75 \text{ N-s} \]

10. When a child falls on a cement floor, his body comes to rest
    instantly. But \( F \times \Delta t = \text{change in momentum} \) is constant.
    As time of stopping \( \Delta t \) decreases, therefore \( F \) increases and hence child
    feels more pain. When he falls on a soft muddy ground in the garden,
    time of stopping increases and hence, \( F \) decreases and he feels lesser pain.

11. Speed of the stream of water, \( v = 15 \) m/s
    Area of cross-section of the tube, \( A = \frac{10^{-2}}{2} \) m\(^2\)
    Volume of water coming out per second from the tube
    \[ V = A \cdot v = \frac{10^{-2} \times 15}{2} = \frac{15 \times 10^{-2}}{2} \text{ m}^3/\text{s} \]
    Density of water = \( 10^3 \) kg/m\(^3\)
    Mass of the water coming out of the tube per second
    \[ m = V \cdot \rho \]
    \[ = \frac{15 \times 10^{-2}}{2} \times \frac{10^3}{2} \text{ kg} = 150 \text{ kg/s} \]
    Force exerted on the wall by the impact of water
    \[ = \text{change in momentum per second} \]
    \[ = m \cdot v = 150 \times 15 \text{N} = 2250 \text{ N} \]

12. Radius of circular bend \( r = 30 \) m
    Speed of the train \( v = 54 \) km/h = \( \frac{54 \times 1}{18} = 3 \) m/s
    \[ = 15 \text{ m/s} \]
    Mass of the train \( m = 10^6 \) kg.
    Let \( \theta \) be the angle of banking required to prevent the wearing out
    the rails, then
    \[ \tan \theta = \frac{v^2}{rg} \]
    \[ = \frac{225}{30 \times 9.8} = \frac{225}{294} = 0.7653 \]
    \[ \theta = \tan^{-1}(0.7653) = 37.5^\circ \]

13. Mass of stone \( m = 0.25 \) kg
    Radius of the string \( r = 1.5 \) m
    Frequency \( v = 40 \) rev/min = \( \frac{40}{60} = \frac{2}{3} \) rev/s
    Centripetal force required for circular motion is obtained from the
    tension in the string.
    \[ T = m \cdot r \cdot \omega^2 = m \cdot r \cdot (2\pi e)^2 \]
    \[ = 0.25 \times 1.5 \times 4 \times \left( \frac{22}{7} \right)^2 \times \left( \frac{2}{3} \right)^2 = 6.6 \text{ N} \]
    Maximum tension which can be withstand by the string,
    \[ T_{\text{max}} = 200 \text{ N} = \frac{mv^2}{r} \]
    \[ v_{\text{max}} = \frac{T_{\text{max}} \cdot r}{m} \]
    \[ = \frac{200 \times 1.5}{0.25} = 34.6 \text{ m/s} \]

14. Let tensions in the string be \( T_1 \) and \( T_2 \) at lowest and highest points
    of the vertical circle.
    At lowest point, \( T_1 \) acts towards the centre of the circle.
    \[ \therefore \quad \text{At the lowest point, string will be stretched downward due to weight of the stone} \]
    and \( mg \) acts vertically downward (weight of an object always acts downward).
    \[ \therefore \quad \text{Net force acting on the stone in upward direction} \]
    \[ T_2 = T_1 - mg \]
    At highest point both \( T_2 \) and \( mg \) act vertically downward. \[ \therefore \quad \text{At the highest point string is stretched away from the centre} \]
    towards the centre of the vertical circle. Net force acting on the stone in the
    upward direction.
    \[ F_2 = T_1 - mg \]
    Thus, the correct answer is (i).
15. When an object is thrown vertically upward or it falls vertically downward under gravity, then an acceleration \( g = 10 \text{ m/s}^2 \) acts downward due to its earth’s gravitational pull.

Mass of the pebble \((m) = 0.05 \text{ kg}\).

(i) **During upward motion** Net force acting on the pebble \( F = ma \) = \(0.05 \times 10 \text{ N} = 0.50 \text{ N} \) (Vertically downward)

(ii) **During downward motion**

Net force acting on the pebble \((F) = m \cdot a\) = \(0.05 \times 10 \text{ N} = 0.50 \text{ N} \) (Vertically downward)

(iii) **At the highest point**

Net force acting on the pebble \((F) = m \cdot a\) = \(0.05 \times 10 \text{ N} = 0.50 \text{ N} \) (Vertically downward)

If pebble was thrown at an angle of 45° with the horizontal direction, then acceleration acting on it and therefore, force acting on it will remain unchanged, i.e., 0.50 N (Vertically downward). In case (iii), at the highest point, the vertical component of velocity will be zero but horizontal component of velocity will not zero.

16. Let the mass of the car be \( M \). The force on the car are

(a) weight \( Mg \) downward

(b) normal force \( N \) by the road upward

(c) friction \( f \), by the road towards the centre.

The car is going on a horizontal circle of radius \( R \). So, it is accelerating the acceleration towards the centre and its magnitude is \( \frac{v^2}{R} \) where \( v \) is the speed.

For vertical direction, acceleration = 0. Resolving the force in vertical and horizontal directions and applying Newton’s laws, we have

\[
N = m \cdot g \quad \text{and} \quad f = M \frac{v^2}{R}
\]

As we are looking for the maximum speed for no skidding, it is a case of limiting friction and hence, \( f = \mu_s \cdot mg \)

So, we have, \( \mu_s \cdot Mg = M \frac{v^2}{R} \rightarrow v^2 = \mu_s \cdot g \cdot R \)

Putting the values, \( v = \sqrt{0.2 \times 10 \times 45} = 9.5 \text{ m/s} \)

Here, the teacher went to teach the student through an easy example. This act of teacher shows the experience and ideality of the teacher.

17. Mass of stone \((m) = 0.1 \text{ kg}\)

(i) When stone is dropped from the window of a stationary train, it falls freely under gravity.

\(. \) Net force acting on stone \((F) = m \cdot g\) = \(0.1 \times 10 \text{ N} = 1.0 \text{ N} \) (Vertically downward)

(ii) Just after the stone is dropped from the window of a train running at a constant velocity, i.e., acceleration of the train is zero. So, no force acts on the stone due to motion falls freely under gravity.

The force acts on it due to its weight only.

(iii) The train accelerates horizontally by \( 1 \text{ m/s}^2 \), but as the stone is left it move under gravity only so the net force on it will be due to gravity only.

(iv) As the stone is accelerating with the train, so there must be a net horizontal force on it.

\( F = m \times a_{\text{train}} = 0.1 \times 1 = 0.1 \text{ N} \)

18. Mass of the body \( A(m_1) = 5\text{ kg} \). Mass of the body \( B(m_2) = 10 \text{ kg} \)

Coefficient of friction between the bodies and the table \( (\mu) = 0.15 \)

Force applied horizontally at \( A; F = 200 \text{ N} \)

(i) **Reaction of the wall** Limiting friction acting to the left

\( f = \mu_R (m_1 + m_2); g = 0.15 (5 + 10); 9.8 = 22.05 \text{ N} \)

\(. \) Net force acting on the wall towards the right

\( F' = F - f = (200 - 22.05) \text{ N} = 177.95 \text{ N} \)

According to Newton’s third law of motion, reaction of the wall = Net force acting on the wall.

\( = 177.95 \text{ N} \) (towards the left)

(ii) **Action-reaction force between \( A \) and \( B \)**

Let \( f \) be the force of limiting friction acting on body \( A \) and \( f \) be the net force applied by body \( A \) on body \( B \).

\[
F = A \quad \text{and} \quad F = B \quad \text{[\( \cdot R = m_1 \cdot g \) \( \text{[towards the left]} \)]}
\]

\( = 0.15 \times 5 \times 9.8 = 7.35 \text{ N} \) (towards the left)

\(. \) Net force applied by body \( A \) on body \( B \)

\( F = (F - f) = (200 - 7.35) = 192.65 \text{ N} \) (towards the right)

According to Newton’s third law of motion, reaction force applied by body \( B \) on body \( A \), i.e.

\( A = f = 192.65 \text{ N} \) (towards the left)

19.\( \sum F_x = Ma; a = \frac{T}{M} \times 18 \text{ N} = 3.60 \text{ m/s}^2 \) to the right.

(ii) \( v = \text{constant}, a = 0 \), so \( F = 0 \) (This is also an equilibrium situation).

(iii) Someone in the car (noninertial observer) claims that the forces on the mass along negative \( x \) are \( T \) and a fictitious force \( (-Ma) \). Someone at rest outside the car (inertial observer) claims that \( T \) is the only force on \( M \) in the positive \( x \)-direction.

20. The car moves to the right with acceleration \( a \). We find the acceleration of \( a_2 \) of the block relative to the earth. The block moves to the right also.

\[
\sum F_x = m_2 a_2; n - m = 0; m = mg, f = \mu_m g
\]

The acceleration of the block relative to the car is \( a_2 = a - \mu_m g \). In this frame the block starts from rest and undergoes displacement \( l \) and gains speed according to

\[ v_f^2 = v_0^2 + 2a_s (x_1 - x_0); v_0^2 = 0 + 2 (\mu_m g - a) (-l - 0) = 2 l \frac{(a - \mu_m g)}{a - \mu_m g} \]

(i) \( v = (2l)(a - \mu_m g))^{1/2} \) to the left (relative to box car)

(ii) The time for which the box slides is given by

\[ \Delta x = \frac{1}{2} (v_s + v_w) t - l = \frac{1}{2} [0 - (2 l (a - \mu_m g))]^{1/2}; t = \left( \frac{2 l}{a - \mu_m g} \right)^{1/2} \]

The car in the Earth frame acquires speed in time \( t \)

\( v_{fr} = v_s + at = 0 + a \left( \frac{2 l}{a - \mu_m g} \right)^{1/2} \) . The speed of the box in the earth frame is then

\[
v_{bo} = v_{co} + v_{ce} = -(2l (a - \mu_m g))^{1/2} + a \left( \frac{2 l}{a - \mu_m g} \right)^{1/2} \]

\[= \left( -2 l \right)^{1/2} \frac{(a - \mu_m g)}{(a - \mu_m g)^2} \left( \frac{a - \mu_m g}{a - \mu_m g} \right)^{1/2} \]

\[= \mu_m g \left( \frac{2 l}{a - \mu_m g} \right)^{1/2} \left( \frac{a - \mu_m g}{a - \mu_m g} \right)^{1/2} \]

\[= \mu_m g \left( \frac{2 l}{a - \mu_m g} \right)^{1/2} \]
MAGNETISM

FOR CLASS XII

Topicwise Collection of best subjective problems

Very Short Answer Type Questions [1 Mark]

1. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?
   [Delhi 2009]

2. Out of the two magnetic materials, 'A' has relative permeability slightly greater than unity while 'B' has less than unity. Identify the nature of the materials 'A' and 'B'. Will their susceptibilities be positive or negative?
   [HOTS; Delhi 2014]

3. What is the angle of dip at a place where the horizontal and vertical components of the Earth’s magnetic field are equal?

4. The permeability of a magnetic material is 0.9983. Name the type of magnetic material it represents.
   [CBSE; Delhi 2011]

5. The susceptibility of a magnetic material is $1.9 \times 10^{-3}$. Name the type of magnetic material it represents.
   [Delhi 2011]

6. The susceptibility of a magnetic material is $-4.2 \times 10^{-6}$. Name the type of magnetic material it represents.
   [HOTS]

Short Answer Type Questions [2 Marks]

7. Define magnetic susceptibility of a material. Name two elements, one having positive susceptibility and the other having negative susceptibility. What does negative susceptibility signify?
   [Delhi 2008]

8. A small compass needle of magnetic moment $m$ is free to turn about an axis perpendicular to the direction of uniform magnetic field $B$. The moment of inertia of the needle about the axis is $I$. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence, deduce the expression for its time period.
   [Ajmer 2009; Delhi 2013]

9. A compass needle, free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (i) horizontal component of earth’s magnetic field and (ii) angle of dip at the place.

10. A magnetised needle of magnetic moment $4.8 \times 10^{-2} \text{JT}^{-1}$ is placed at $30^\circ$ with the direction of uniform magnetic field of magnitude $3 \times 10^{-2} \text{T}$. Calculate the torque acting on the needle.
   [Foreign 2012]

11. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip down at $60^\circ$ with the horizontal. The horizontal component of the earth’s magnetic field at the place is known to be $0.4 \text{ G}$. Determine the magnitude of the earth’s magnetic field at the place.

12. A magnet was found to vibrate at a place with a time period to $T$. A piece of brass of same length, breadth and mass was placed over the magnet. What will be the new time period?

13. A hypothetical bar magnet $AB$ is cut into two equal parts. One part is now kept over the other as shown in the figure.

If $M$ is the magnetic moment of the original magnet, what would be the magnetic moment of the following combinations so formed?

- (i) $N$ $S$
- (ii) $N$ $S$ $S$ $N$
14. A hypothetical bar magnet $AB$ is cut into two equal parts. One part is now kept near/over the other as shown in the figure. If $M$ is the magnetic moment of the original magnet, what would be the magnetic moment of the following combinations so formed?

$\begin{array}{cc}
\text{(i)} & \text{(ii)} \\
N & S \\
N & S \\
(S & N \\
\end{array}$

15. Identify the magnetic materials from the following curves.

$\begin{array}{cc}
\text{(i)} & \text{(ii)} \\
I & I \\
M & M \\
\end{array}$

16. (a) If the magnetic monopoles were to exist, how would the Gauss’ law of magnetism get modified?

(b) How will the angle of dip vary when one goes from a place, where the acceleration due to gravity is maximum, to a place where it is minimum on the surface of earth?

17. Two magnets of magnetic moments $M$ and $M\sqrt{3}$ are joined to form a cross. The combination is suspended in a uniform magnetic field $B$. The magnetic moment $M$ now makes an angle of $\theta$ with the field direction. Find the value of angle $\theta$.

$\begin{array}{cc}
\text{M} & \sqrt{3} \\
\text{M} \\
\end{array}$

18. What is the basic difference between the atom or molecule of a diamagnetic and a paramagnetic material? Why are elements with even atomic number more likely to be diamagnetic?

**Short Answer Type Questions [3 Marks]**

19. A current in a solenoid having air in the interior creates a magnetic field $B = \mu_0 H$. Describe qualitatively what happens to the magnitude of $B$ as (a) aluminium (b) copper and (c) iron are placed in the interior.

20. What are permanent magnets? What is an efficient way of preparing a permanent magnet? Write two characteristic properties of materials which are required to select them for permanent magnets. [AI 2008]

21. (i) What happens when a diamagnetic substance is placed in a varying magnetic field?

(ii) Name the properties of a magnetic material that make it suitable for making (a) a permanent magnet and (b) a core of an electromagnet.

22. (i) How does angle of dip change as one goes from magnetic pole to magnetic equator of the earth?

(ii) A uniform magnetic field gets modified as shown below when two specimens X and Y are placed in it. Identify whether specimens X and Y are diamagnetic, paramagnetic or ferromagnetic. [Foreign 2009]

(iii) How is the magnetic permeability of specimen X different from that of specimen Y?

23. Identify the magnetic material which follows the graphical representation given below. Also locate any two inferences about the behaviour of the material from the graph. [HOTS]

**Value Based Questions [3 Marks]**

24. Two girls Pooja and Ritu were very good dance and performed in the school function using their cassette player. One day when they were performing, tape got stuck up and the music stopped. But Pooja was determined not to be let down the performance, so she sang the song instead of dancing and Ritu completed the dance.

(i) What were the values displayed by Pooja and Ritu?
(ii) What kind of ferromagnetic material is being used for coating magnetic tapes used in cassette players or building memories stories in modern computers?

(iii) What is the value of susceptibility of a ferromagnetic substance.

25. Prachi’s mother had put lot of clothes for washing in the washing machine, but the machine did not start and an indicator was showing that the lid did not close. Prachi seeing her mother disturbed thought that she would close the lid by force that but realised that the mechanism was different. It was a magnetic system. She went to the shop and got a small magnetic door closer and put in on the lid. The machine started working. Her mother was happy that Prachi helped her to save ₹ 300/- also.

(i) What was the value developed by Prachi?
(ii) What values did her mother impart to Prachi?
(iii) Every magnetic configuration has a north pole and a south pole. What about the field due to toroid.

### Solutions

1. Magnetic field lines can be entirely confined within the core of a toroid because toroid has no ends. A solenoid in open ended and the field lines inside it which is parallel to the length of the solenoid, cannot form closed curved inside the solenoid.

2. A → Paramagnetic  \( (\cdot \mu, > 1) \)
   - B → Diamagnetic  \( (\cdot \mu, < 1) \)
   - Susceptibility for A → Positive
   - Susceptibility for B → Negative

3. We know, \( \frac{B_N}{B_N} = \tan \delta \)

   Given \( B_Y = B_N \), then \( \tan \delta = 1 \)
   - Angle of dip, \( \delta = 45^\circ \)

4. \( \mu, < 1 \), so magnetic material is diamagnetic.

5. Susceptibility is small positive, so material is paramagnetic.

6. Susceptibility of material is negative, so given material is diamagnetic.

7. It is defined as the intensity of magnetisation per unit magnetising field.
   - \( \chi = \frac{M}{H} \). It has no unit.
   - Iron has positive susceptibility while copper has negative susceptibility.
   - Negative susceptibility of a substance signifies that the substance will be repelled by a strong magnet or opposite feeble magnetism induced in the substance.

8. If magnetic compass of dipole moment \( m \) is placed at angle \( \theta \) in uniform magnetic field and released it experiences a restoring torque \( \tau \).
   \[ \tau = - m \times B \]

where, \( m = \text{pole strength} \)

\[ |\tau| = m |B| \sin \theta \]

In equilibrium, the equation of motion,

\[ \frac{d^2 \theta}{dt^2} = -\frac{|m| |B| \theta}{l} \]  \( \text{(For small angle, } \sin \theta \approx \theta \) \)

\[ \frac{d^2 \theta}{dt^2} = -\frac{|M| |B| \theta}{l} \]

\[ \frac{d^2 \theta}{dt^2} = -\theta \]

Since,

\[ \frac{\theta}{\theta} = -\theta \]

It represents the simple harmonic motion with angular frequency

\[ \omega^2 = \frac{|M| |B|}{l} \]

\[ T = \frac{2 \pi}{\omega} = 2 \pi \sqrt{\frac{l}{MB}} \]

9. If compass needle orients itself with its axis vertical at a place, then

\[ \begin{align*}
  B_N & \quad \delta \quad l \quad B_Y \\
  B & \quad mB \quad S \quad N
\end{align*} \]

\( i) \ B_N = 0 \text{ because } B_Y = |B| \)

\( ii) \ \text{Angle of dip, } \delta = 45^\circ \)

\[ \tan \delta = \frac{B_Y}{B_N} \rightarrow \]

\[ \delta = 45^\circ \]

10. We have,

\[ \tau = MB \sin \theta \]

where, \( \tau = \text{torque acting on magnetic needle} \)

\( M = \text{magnetic moment} \)

\( B = \text{magnetic field strength} \)

then,

\[ \tau = 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \sin 30^\circ \]

\[ = 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \frac{1}{2} \]

\[ \tau = 7.2 \times 10^{-4} \text{ Nm} \]
11. Angle of dip, $\theta = 60^\circ$

\[ H = 0.4 \text{ G} = 0.4 \times 10^{-4} \text{ T} \]

If $B_e$ is earth's magnetic field, then $H = B_e \cos \theta$.

\[ \Rightarrow B_e = \frac{H}{\cos \theta} = \frac{0.4 \times 10^{-4} \text{ T}}{\cos 60^\circ} \]

\[ = \frac{0.4 \times 10^{-4} \text{ T}}{0.5} \]

\[ = 0.8 \times 10^{-4} \text{ T} = 0.8 \text{ G} \]

12. \[ T = 2\pi \sqrt{\frac{I}{MB}} \]

Due to brass piece moment of inertia of oscillator increases to $2I$ but magnetic dipole moment remains same. So, new time period,

\[ T' = 2\pi \sqrt{\frac{I'}{MB}} \]

\[ = 2\pi \sqrt{\frac{2I}{MB}} = \sqrt{2}T \]

13. When magnet is cut to half, each part is a magnet of dipole moment,

\[ M' = \frac{M}{2} \]

In case (i),

\[ M'_{\text{net}} = \frac{M}{2} + \frac{M}{2} = M \]

In case (ii),

\[ M'_{\text{net}} = \frac{M}{2} - \frac{M}{2} = 0 \]

14. When magnet is cut to half (lengthwise) each part is a magnet of moment, $M' = \frac{M}{2}$.

So, net magnetic moment of combination is

(i) $M'_{\text{net}} = \frac{M}{2} + \frac{M}{2} = M$

(ii) $M'_{\text{net}} = \frac{M}{2} - \frac{M}{2} = 0$

(iii) $M'_{\text{net}} = \frac{M}{2} + \frac{M}{2} = M$

15. (i) $M$ increases and then becomes constant.

\[ \therefore \text{Sample is para or ferromagnetic.} \]

(ii) $M$ decreases and is negative.

\[ \therefore \text{Sample is diamagnetic.} \]

16. (a) \[ \oint \mathbf{J} \cdot d\mathbf{s} = \mu_0 \mathbf{M}_m \]

where, $\mathbf{q_m}$ is the net (magnetic) charge (or monopole strength) enclosed by the closed surface.

(b) From $90^\circ$ to $0^\circ$. ($q_{\text{max}}$ at poles)

17. \[ MB \sin \theta = \sqrt{3} MB \sin(90^\circ - \theta) \]

\[ = \sqrt{3} MB \cos \theta \]

\[ \Rightarrow \tan \theta = \sqrt{3} \]

i.e. \[ \theta = 60^\circ \]

18. The atom/molecule of a diamagnetic material has zero net magnetic moment. For a paramagnetic material it is not so. With an even atomic number, the electrons in an atom of an element can 'pair off' which can makes the net magnetic moment of each pair as zero. This makes the element more likely to be diamagnetic.

19. (a) $B$ increases slightly

(b) $B$ decreases slightly

(c) $B$ increases significantly

When each metal is in the solenoid, the total field is

$B = \mu_0 \chi H$. $B$ is slightly greater than $\mu_0 H$ for aluminium and slightly less for copper. For iron, the field can be made thousands of time stronger.

20. Permanent Magnets: The magnets prepared from ferromagnetic materials which retain their magnetic properties for a long time are called permanent magnets.

An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetises the rod.

The materials used for permanent magnet must have the following characteristic properties.

(i) High retentivity so that the magnet may cause strong magnetic field.

(ii) High coercivity so that the magnetisation is not wiped out by strong external fields, mechanical ill-treatment and temperature change. The loss due to hysteresis is immaterial because the magnet in this case is never put to cyclic changes.

21. (i) A diamagnetic substance is attracted towards a region of weaker magnetic field.

(ii) Permanent magnets are made of steel which is characterised by high retentivity and high coercivity.

(b) Electromagnets are made of soft iron which is characterised by high retentivity and low coercivity.

22. (i) Angle of dip decreases from $90^\circ$ to $0^\circ$ as one goes from magnetic pole to magnetic equator of earth.

(ii) $X$ is diamagnetic and $Y$ is ferromagnetic.

(iii) Permeability of $X$ is between 0 to 1 while that of $Y$ is around 1000.

23. Given substance is diamagnetic and it shows superconductivity when cooled below a certain "critical temperature".

From graph we conclude that

(i) $R \to 0$ below a fixed temperature.

(ii) When temperature is 4.2 K, material becomes paramagnetic and its resistance is zero.

24. (i) Pooja and Ritu displayed "Determination" and "Devotion" to their dance performance.

(ii) To coat magnetic tapes with "soft" ferromagnetic substance like soft iron alloy is used.

(iii) Susceptibility for a ferromagnetic substance is around 900.

25. (i) Prachi shows "concern" for her mother.

(ii) Prachi mother felt "proud" of her.

(iii) Toroidal's magnetic field is confined inside its core, it has no south or north pole. Magnetic dipole moment of toroid is also zero.
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Multiple Choice Questions Type I

1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one can ignores the magnetic forces of one on another. This is because [Page 38, Q.1]
   (a) the two magnetic forces are equal and opposite, so they produce no net effect
   (b) the magnetic forces do not work on each particle
   (c) the magnetic forces do equal and opposite (but non-zero) work on each particle
   (d) the magnetic forces are necessarily negligible

2. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one each track as shown in figure. Which of the following statement is correct? [Page 39, Q.7]
   (a) Both the stones reaches the bottom at the same time but not with the same speed
   (b) Both the stones reaches the bottom with the same speed and stone I reaches the bottom earlier than stone II
   (c) Both the stones reaches the bottom with the same speed and stone II reaches the bottom earlier than stone I
   (d) Both the stones reaches the bottom at different times and with different speeds

3. A raindrop falling from a height \( h \) above ground, attains a near terminal velocity when it has fallen through a height \( (3/4)h \). Which of the diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground? [Page 42, Q.15]

4. Two blocks of masses \( M_1 \) and \( M_2 \) having equal mass are free to move on a horizontal plane frictionless surface. \( M_2 \) is attached to a massless spring as shown in figure. Initially, \( M_2 \) is at rest and \( M_1 \) is moving towards \( M_2 \) with speed \( v \) and collides head on with \( M_2 \). [Page 44, Q.21]
   (a) While spring is fully compressed, all the KE of \( M_1 \) is stored as PE of spring
   (b) While spring is fully compressed, the system momentum is not conserved, but final momentum is equal to initial momentum
   (c) If spring is massless, then final state of \( M_1 \) is state of rest
   (d) If the surface on which blocks are moving has friction, then collision cannot be elastic

5. Which of the following diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it in a terminal velocity? [Page 43, Q.17]
EXTRA DOSE

Very Short Answer Type Questions

6. A rough inclined plane is placed on a cart moving with a constant velocity \( u \) on a horizontal ground. A block of mass \( M \) rests on the incline. Is any work done by force of friction between the block and incline? Is there a dissipation of energy? [Page 45, Q.22]

7. Two bodies of unequal masses are moving in the same direction with equal kinetic energy. The two bodies are bought to rest applying retard in force of same magnitude. How would the distance moved by them before coming to rest compare? [Page 46, Q.32]

8. A bob of mass \( m \) suspended by a light string of the length \( L \) is whirled into a vertical circle as shown in the figures. What will be the trajectory of the particle if the string is cut at

(a) point \( B \)  
(b) point \( C \)  
(c) point \( X \)? [Page 46, Q.33]

9. A body falls towards the earth in air. Will its total mechanical energy be conserved during the fall? [Page 46, Q.26]

Long Answer Type Questions

11. A raindrop of mass 1.00 g falling from a height of 1km hits the ground with a speed of 50 ms\(^{-1}\). Calculate

(a) the loss of PE of the drop
(b) the gain in KE of the drop
(c) is the gain in KE equal to loss of PE? If not why? (Take \( g = 10 \text{ ms}^{-2} \))

12. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 50000 kg is moving with a speed of 36 km/h when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, then calculate the spring constant. [Page 47, Q.41]

13. An adult weighing 600 N raises the centre of gravity of his body by 0.25 m while taking each step of 1m in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming, that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food. Calculate the energy equivalent of food that would be required to compensate for energy utilised for jogging. [Page 47, Q.42]

Short Answer Type Questions

10. A ball of mass \( m \), moving with a speed \( 2v_0 \), collides inelastically \((e > 0)\) with an identical ball at rest. Show that

(a) for head on collision, both the balls move forward.
(b) for a general collision, the angle between the two velocities of scattered balls is less than 90\(^\circ\).

14. A block of mass 1 kg is pushed up a surface inclined to the horizontal at an angle of 30\(^\circ\) by a force of 10 N parallel to the inclined surface. The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline. Calculate [Page 48, Q.44]

(a) work done against gravity
(b) work done against force of friction
(c) increase in potential energy
(d) increase in kinetic energy

15. A rocket accelerates straight up by ejecting gas downwards. In a small time interval \( \Delta t \), it ejects a gas of mass \( \Delta m \) at a relative speed \( u \). Calculate the KE of the entire system at \( t + \Delta t \) and show that the device that ejects does work \( \frac{1}{2} \Delta m \cdot u^2 \) in this time interval (neglect gravity). [Page 49, Q.46]
1. (b) As, the magnetic forces due to motion of electron and proton act in a direction perpendicular to the direction of motion, no work is done by these forces. That is, why one ignores the magnetic force of one particle on another.

2. (c) As both surfaces I and II are frictionless and two stones are slide from rest from the same height, therefore both the stones reach the bottom with same speed \( \frac{1}{2}mv^2 = mgh \). As acceleration down plane II is large \( a_2 = g \cdot \sin \theta_2 \) is greater than \( a_1 = g \cdot \sin \theta_1 \), therefore stone II reaches the bottom earlier than stone I.

3. (b) At a height \( h \) above the ground, PE of raindrop is maximum and KE = 0. As, the raindrop falls, its PE goes on decreasing and KE goes on increasing up to a point \( \frac{h}{4} \) above the ground. At this stage, raindrop has acquired near terminal velocity (=constant), PE becomes zero when raindrop falls to the ground. Choice (b) is most appropriate.

4. (c,d) While spring is fully compressed, the entire KE of \( M_1 \) is not stored as PE of spring as \( M_2 \) may move. If spring is massless, then as \( M_1 = M_2 \), velocities of \( M_1 \) and \( M_2 \) are interchanged on collision. \( M_1 \) comes to rest, instead of \( M_2 \). Choice (c) is correct.

If surface on which blocks are moving has friction, loss of energy is involved. Collision cannot be elastic. Choice (d) is correct.

5. (b) When an iron sphere falls freely in a lake, its motion is accelerated due to gravity and retarded due to viscous force. The overall effect is increase in velocity and hence, increase in KE till the sphere acquires terminal velocity which is constant. Hence, KE of sphere beyond this depth of lake becomes constant. Option (b) is correct.

6. The force of friction between the block and the incline prevents the block from sliding down. As, the block not move, therefore, no work is done by the force of friction.

7. Work done in stopping the body = Force \times Distance = KE of body, which is same for two bodies.

   As retarding force applied is same, therefore distance by which the bodies before coming to rest must be the same.

8. When the string is cut, centripetal force is no longer provided. The bob flies along the tangent to the arc at that point.

   (a) When string is cut at B, the bob moves vertically downwards.

   (b) When string is cut at C, initial velocity of bob is along the horizontal to the right as shown in Fig. (i). Therefore, the bob follows a parabolic path which vertex at C.

   (c) When the string is cut at X, initial velocity is along tangent to the circle at X. The bob will follow a parabolic path with vertex higher than C.

9. No, this is because some work has to be done against resistive force of air which is non-conservative force. Therefore, gain in KE < Loss in PE. Hence, mechanical energy is not conserved.

10. (a) If \( v_1, v_2 \) are velocities of two balls after collision, then according to the principle of conservation of linear momentum,

\[
m(2v_0) = m - v_1 + m - v_2
\]

or

\[
2v_0 = v_1 + v_2 \tag{1}
\]

By definition, \( e = \frac{v_2 - v_1}{v_0} \), \( v_2 - v_1 = v_2 + v_1 \)

\[
\therefore \quad v_2 = v_1 + 2v_0 \cdot e \tag{2}
\]

From Eq. (i), \( 2v_0 = v_1 + (v_1 + 2v_0 \cdot e) \)

\[
= 2(v_1 + v_0 \cdot e)
\]

\[
\Rightarrow \quad v_1 = v_0(1 - e).
\]

As, \( e < 1, v_1 \) has the same sign as \( v_0 \).

\( \therefore \) The ball moves forward after collision.

From Eq. (ii), \( v_2 = v_0(1 - e) + 2v_0 \cdot e = v_0(1 + e) \)

Again, as, \( e < 1, v_2 \) has the same sign as \( v_0 \). Hence, the other ball moves forward after collision.

(b) According to the "principle of conservation of linear momentum",

\[
p = p_1 + p_2
\]

For a general collision, some KE is lost.

\[
\Rightarrow \quad \frac{p_1^2}{2m} > \frac{p_2^2}{2m}
\]

or \( p_1^2 > (p_2^2 + p_0^2) \).

Which is possible only when \( \theta < 90^\circ \), which was to be proved.

11. Here, mass (\( m \)) = 1.00 g = 10\(^{-3}\) kg

Height (\( h \)) = 1 km = 10\(^3\) m

Velocity (\( v \)) = 50 m/s

\( g = 10 \text{ m/s}^2 \)

(a) Loss of PE of the drop

\[
= mgh = 10^{-3}\cdot(10\cdot10^3) = 10 \text{ J}
\]

(b) Gain in KE of the drop

\[
= \frac{1}{2}mv^2 = \frac{1}{2}\cdot10^{-3}\cdot(50)^2 = 1.25 \text{ J}
\]

(c) No, this is because a part of PE lost is used up is doing work against the viscous drag of the air.

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12. Here, \( l = 15 \text{ m}, m = 50000 \text{ kg} = 5 \times 10^4 \text{ kg} \)
\[ v = 36 \text{ km/h} = \frac{36 \times 1000}{60 \times 60} \text{ m/s} = 10 \text{ m/s} \]
\[ x = 1.0 \text{ m}, k = ? \]
Total KE
\[ \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times 10^4 \times (10)^2 \]
\[ = 2.5 \times 10^6 \text{ J} \]
As 90% of energy of wagon is lost due to friction, therefore energy transferred to shock absorber,
\[ E = \frac{10}{100} \times 2.5 \times 10^6 = 2.5 \times 10^5 \text{ J} \]
From \( E = \frac{1}{2} kx^2 \),
\[ k = \frac{2E}{x^2} = \frac{2 \times 2.5 \times 10^5}{(10)^2} \]
\[ = 5 \times 10^5 \text{ N/m} \]
13. Here,
\[ mg = 600 \text{ N} \]
\[ h = 0.25 \text{ m per step.} \]
Number of steps
\[ n = \frac{6 \times 10^3}{1} = 6 \times 10^3 \]
Energy utilised in jogging
\[ = 6 \times 10^3 \times 600 \times 0.25 \text{ J} \]
\[ E = 9 \times 10^5 \text{ J} \]
This is 10% of intake.
\[ \therefore \text{ Intake energy} = 10E = 9 \times 10^6 \text{ J} \]
14. In the figure, \( f \) is the force of friction acting down the inclined plane.
Here,
\[ m = 1 \text{ kg}; \theta = 30^\circ \]
\[ F = 10 \text{ N}; \mu = 0.1; s = 10 \text{ m} \]
(a) Work done against gravity
\[ W_g = mg \sin \theta \times s = 1 \times 10 \sin 30^\circ \times 10 \]
\[ = \frac{100}{2} = 50 \text{ J} \]
(b) Work done against friction
\[ W_f = f \times s = \mu N \times s = \mu \cdot mg \cos \theta \times s \]
\[ = 0.1 \times 1 \times 10 \cos 30^\circ \times 10 \]
\[ = 10 \times 0.886 = 8.86 \text{ J} \]
(c) Increase in PE
\[ = mgh = mg (s \cdot \sin \theta) \]
\[ = 1 \times 10 \times 10 \sin 30^\circ = 50 \text{ J} \]
(d) Increase in KE, \( \Delta K = ? \)
Acceleration of block up the incline
\[ a = \frac{F - (mg \sin \theta + \mu \cdot mg \cos \theta)}{m} \]
\[ = 10 - \left(1 \times 10 \times \frac{1}{2} + 0.1 \times 10 \times 0.866 \right) \]
\[ = \frac{1}{1} = 4.13 \text{ m/s}^2 \]
From equation of motion,
\[ v^2 - u^2 = 2as \]
\[ \Delta K = \frac{1}{2} m(v^2 - u^2) = \frac{1}{2} m \times 2 \text{ as} = m \Delta \text{u} \]
\[ = 1 \times 413 \times 10 = 413 \text{ J} \]
15. Let \( M \) be the mass of the rocket at any time \( t \) and \( v \), be the velocity of rocket at this time.
\( \Delta m \) = mass of gas ejected in time interval \( \Delta t \) with relative velocity \( u \).
\[ \therefore \text{ At time} (t + \Delta t), (KE) = E_2 = (KE \text{ of rocket} + KE \text{ of gas}) \]
\[ = \frac{1}{2} (M - \Delta m) (v + \Delta u)^2 = \frac{1}{2} (M - \Delta m) (v^2 + \Delta u^2) \]
Expanding and neglecting terms, which are too small, we get
\[ E_2 = \frac{1}{2} Mv^2 + M \cdot v \cdot \Delta u - (\Delta m) \cdot v \cdot u + \frac{1}{2} (\Delta m) \cdot u^2 \]
\[ (E_2 - E_1) = [M \cdot \Delta u - (\Delta m) \cdot u] v + \frac{1}{2} (\Delta m) \cdot u^2 \]
As,
\[ M \frac{dv}{dt} = \frac{d}{dt} [M \Delta u - (\Delta m) u] = 0 \]
From Eq (i),
\[ E_2 - E_1 = \frac{1}{2} (\Delta m)(u^2) = \Delta \text{E} \]
using work-energy theorem.

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When helium is cooled to almost absolute zero (−460°F or 273°C), the lowest temperature possible, it becomes a liquid with surprising properties, it flows against gravity and will start running up, why?

When helium is just a few degrees below its boiling point of −452 degrees Fahrenheit (−269 degrees Celsius) it will suddenly be able to do things that other fluids can’t, like dribble through molecule thin cracks, climb up and over the sides of a dish and remain motionless when its container is spun. No longer a mere liquid, the helium has become a super fluid—a liquid that flows without friction. Atoms in the liquid will collide with one another and slow down. But if we did that with helium at low temperature and came back a million years later, it would still be moving.

Do you know, Venus is the only planet to spin clockwise? Explain.

Our solar system started off as a swirling cloud of dust and gas which eventually collapsed into a spinning disc with the Sun at its centre because of this common origin, all the planets move around the Sun in the same direction and on roughly the same plane. They also all spin in the same direction (counter clockwise if observed from above) — except Uranus and Venus. Uranus spins on its side, while Venus defiantly spins in the complete opposite direction. The most likely cause of these planetary oddballs are gigantic asteroids which knocked them off course in the distant past.

Have you ever heard about the technology behind “Underwater Wireless Communication”? Do you know?

Underwater wireless communication is a flourishing research area in the field of wireless communications. It represents the overall framework of the necessity of underwater wireless systems, characteristics of an acoustic channel, hardware and working of acoustic modems, sensor networks and different communication architectures involved in the sensor networks. Applications till date, like oceanographic data collection, AUVs (Autonomous Underwater Vehicles), underwater radio, etc., future challenges like effective transmission of video and audio signals by real time monitoring have been emphasised with a view to overcome the present limitations.

Neutron stars are the fastest spinning objects known in the universe. Do you know?

Neutron stars are thought to be the fastest spinning objects in the universe. Pulsars are a particular type of neutron star that emits a beam of radiation which can be observed as a pulse of light as the star spins. The rate of this pulse allows astronomers to measure the rotation. The fastest spinning known pulsar is the catchily-titled PSR J1748-2446ad, which has an equator spinning at 24% the speed of light, which translates to over 70000 kilometres per second.

A research works proved magnesium phosphide (MnP) as an unexpected superconductor. Do you know?

Researchers have discovered the first manganese based superconductor, a compound whose strong magnetism was thought to prevent superconductivity Researches in China have now discovered that manganese phosphide (MnP) can be made superconducting by application of a large pressure. Superconductivity in MnP seems to be related to its exotic magnetic structure, based on a “helical” arrangement of spins. The material is superconducting only at very low temperatures and under pressure, but its discovery suggest higher temperature superconductivity could be found in other helical magnets.

Do you know, antimatter is a composition of antiparticles that have same mass but with opposite charge and spin?

All matter in the universe is built up from a relatively small number of elementary particles, which include quarks (the constituents of protons and neutrons) and electrons (which together with protons and neutrons, make up atoms). Associated with each elementary particle is an ‘antiparticle’ which also occurs in nature. The antiparticle has the same mass as the particle but with opposite charge. A particle and an antiparticle can combine (or “annihilate”) to produce a photon or a particle of light. Conversely, a particle-antiparticle pair can be produced from a photon. So there is a type of symmetry between particles and antiparticles in these processes. More specifically, the subatomic particles of antimatter have properties opposite those of normal matter. The electrical charge of those particles is reversed.

Do you know, on an average a human body carries ten times more bacterial cells than human cells?

For one thing, bacteria produce chemicals that help us harness energy and nutrients from our food. Germ free rodents have to consume nearly a third more calories than normal rodents to maintain their body weight and when the same animals were later given a dose of bacteria, their body fat levels spiked, even if they didn’t eat any more than they had before. The gut bacteria is also very important to maintaining immunity.

Do you know the fact, an individual blood cell takes about 60 seconds to make a complete circuit of the body? Explain.

We have about 5 litres of blood in your body and the average heart pumps about 70 mL of blood out with each beat. Also, a healthy heart beats around 70 times a minute. So, if we multiply the amount of blood that the heart can pump by the number of beats in a minute, we actually get about 4.9 litres of blood, which is almost our whole body’s worth of blood. In just a minute, the hearts pumps the entire blood volume around our body.
While doing practice of chapter Motion in a Plane, I was able to understand all of its concepts but I am not able to figure it out that a glass marble slides from rest of the topmost point of a vertical circle of radius $r$ along a smooth chord, does the time of descent depend upon the chord chosen?

[Shalu Yadav, Delhi]

- Let us check this, suppose $C$ be the centre of a circle. Suppose chord $AB$ makes angle $\theta$ with the vertical. If the marble takes time $t$ to slide along chord $AB$, then

$$AB = \sqrt{r^2 + \left(\frac{r}{\sin \theta}\right)^2}$$

$$\therefore \quad t = 2 \times \frac{r}{\sin \theta} \sqrt{\frac{r}{g}}$$

As, $t$ is independent of $\theta$, so the time of descent does not depend on the chord chosen.

Sir, the chapter Electronic Device is being one of the most important chapter from the examination point of view. I covered almost all of its concept except one, i.e. the ionisation energy of isolated pentavalent phosphorous atom is very large. How is it possible that when it goes into silicon crystal it releases its fifth electron at room temperature, so that $n$-type semiconductor is obtained? [Sathyam, Trivanantapuram]

- When a phosphorous atom of five valence electrons are doped into silicon crystal, the four valence electrons of phosphorous atom will form covalent bonds by sharing electrons with the neighbouring four atoms of silicon.

The fifth valence electron is loosely bound as it requires small energy to become a free electron within the crystal, which is explained below.

According to Bohr’s theory in an alone hydrogen atom, the energy of electron in $n$th orbit is

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

when $n = 1$, then $E = \frac{-13.6}{1^2} = -13.6 \text{ eV}$

... (i)
When hydrogen atom is surrounded by a medium of dielectric constant $K$, then from Eq. (i), we get

$$E_n = -\frac{13.6}{K^2} \text{ eV}$$

In case of Si, $K = 12$, then

$$E_n = -\frac{13.6}{12^2} \text{ eV} = -0.1 \text{ eV}$$

It means the ionisation energy is 0.1 eV. That is why the fifth electron of phosphorous is loosely bound to the ion core. So, we get an n-type semiconductor when Si is doped with P.

While doing practice of chapter Photocell Effect and Wave Particle Duality, I don’t get it how to check this numerical problem i.e. a monochromatic light of wavelength $\lambda$ is incident on an isolated metallic sphere of radius $a$. The threshold wavelength is $\lambda_{th}$, which is larger than $\lambda$. Calculate the number of photoelectrons emitted before the emission of photoelectrons will stop.

[Tusli Jawdekar, Mumbai]

- As we know, the metallic sphere is isolated, it becomes positively charged when electrons are ejected from it. There is an extra attractive force on the photoelectrons. If the potential of the sphere is raised to $V$, the electron should have a minimum energy $(\phi) + eV$ to be able to come out. Thus, emission of photoelectrons will stop, when

$$\frac{hc}{\lambda} = (\phi) + eV = \frac{hc}{\lambda_{th}} + eV$$

So, stopping potential of a photoelectron,

$$V = \frac{hc}{\phi} \left( \frac{1}{\lambda} - \frac{1}{\lambda_{th}} \right)$$

The charge on the sphere needed to take its potential to $V$ is

$$Q = (4\pi\varepsilon_0 a) V$$

The number of electrons emitted from metallic sphere,

$$n = \frac{Q}{e} = \frac{4\pi\varepsilon_0 a^2 V}{e}$$

While going through chapter-Thermodynamics, I was quite confident regarding all of its concepts except one conceptual problem, where I got stuck, i.e. the search for a perpetual motion machine has kept inventors occupied for centuries. Comment on this statement.

[Shalini Sethi, Mohali]

- For centuries, scientists have been trying to fabricate machines which can generate perpetual motion means they run by themselves without any energy from outside. The statement is perfectly true. However, their attempts have not met with success so far. Theoretically, there can generally be three types of perpetual motion machines.

  (i) A mechanism which would produce machines with efficiency more than 100%. This would violate first law of thermodynamics.

  (ii) A device that would extract heat from a source and convert it completely into other forms of energy, e.g., an ocean linear that uses heat from the ocean for propulsion. This would violate the second law of thermodynamics.

  (iii) A mechanism by which energy dissipation can be eliminated cent per cent. This again go against Lind lane.

While doing practice of chapter-Work, Energy and Power, I covered all of its concepts, but I was not able to understand the concept of law of conservation of mechanical energy. Please, explain the reason in brief. [Prachi Desai, Pune]

- Suppose only conservative force operates on a system of particles and potential energy $U$ is defined corresponding to these factors. There are either no other forces or work done by them is zero. We have

$$U_i - U_f = W$$

$$W = K_i - K_f$$

[As, work-energy theorem]

$$U_i - U_f = -(K_i - K_f)$$

$$\Rightarrow U_i + K_i = U_f + K_f$$

$(i)$

The sum of the potential and kinetic energy is called total mechanical energy. We see from Eq. (i) that the total mechanical energy of a system remains constant, if only conservative forces are acting on a system of particles and work done by all other forces is zero. This is called conservation of mechanical energy. The total mechanical energy is not constant, if non-conservative force such as friction is acting between the parts of a system. However, the work-energy theorem is still valid. Thus, we can apply,

$$W_i + W_{nc} + W_{re} = K_i - K_f$$

Here,

$$W_i = (U_i - U_f)$$

So, we get

$$W_{nc} + W_{re} = (K_i + U_i) - (K_f + U_f)$$

or

$$W_{nc} + W_{re} = E_i - E_f$$

Here, $E = K + U$ is total mechanical energy.

While going through the chapter-Magnetic Effect of Current and Magnetism, I was quite confident regarding all of its concepts but on one conceptual problem I found myself quite confused, i.e. what role does magnetism play in ground control of artificial satellites? Please, explain the reason in brief. [Shabnam Raza, Faridabad]

- Artificial satellites are often provided with loops of wire called torque coils. These coils can be activated by a computer or a satellite operator on earth. On activation, a suitable current is made to flow through a particular torque coil. This current carrying coil behaves like a magnet. As earth is behaving like a huge magnet by itself, a magnetic force is exerted by the earth on the satellite. As a result, the satellite can be oriented so that its instrument point in the desired direction. A magnetometer on the artificial satellite senses the direction of earth’s magnetic field. The satellite ground controls use the feedback from this device to decide which torque coil to activate.

**Note** In order to protect other equipment on artificial satellites, the body of the satellite must remain unmagnetised, even when the torque coils have been activated. This can be done by selecting suitable materials for building the rest of the satellite.

Sir, when I was going to read the chapter-Gravitation, we know that, both the moon and the sun produce our ocean tides. We also know that moon plays the greater role, because it is closer. Does its closeness mean it pulls with more gravitational force than the sun on the earth’s oceans? [Sandeep Sharma, Mumbai]

- Yes of course although sun is more massive ($F \propto M$) but it is also more far

$$F \propto \frac{1}{r^2}$$

So, moon’s influence is more than the sun. Also, when moon and sun are on same side we get much higher tides.

$$E \rightarrow m \rightarrow S \rightarrow$$

**Much higher tide**
1. A uniform ball of radius $r$ rolls without slipping from the top of a sphere of radius $(R > r)$. Find the angular velocity of the ball at the moment it breaks off the sphere. The initial velocity of the ball is negligible.

2. A smooth sphere is supported in contact with a smooth vertical wall by a string fastened to a point on its surface, the other end being attached to a point in the wall; if the length of the string be equal to the radius of the sphere. Find the inclination of the string to the vertical, the tension of the string and the reaction of the wall.

3. Two blocks of masses $m_1$ and $m_2$ connected by an undeformed massless spring rests on a horizontal plane.

Find the minimum constant force $F$, that has to be applied in the horizontal direction to the block of mass $m_2$ so that the other block gets shifted. Let $\mu$ be the coefficient of friction between blocks and surface.

4. Consider the system of blocks shown below:

Determine the apparent weight of block 2.

Assume that friction between block 1 and block 2 is high enough so that the block 2 remains stationary at rest with respect to block 1.

Also assume that the inclined plane does not move.

5. A class XIIth student designed a system to determine coefficient of friction.

It consists of 3 blocks $A$, $B$ and $C$ connected by the help of ideal pulleys and ideal strings as shown in figure below:

Mass of block $C = $ mass of block $B = m$ and mass of block $A = 3m$.

When he release block $C$ from rest, it is found that blocks $A$ and $B$ does not moves. If all surfaces are made up of same material having same surface finish, find range of coefficient of friction.

6. A boy dropped a ‘bouncy’ ball over an incline plane from a height $H$. Ball strikes the incline plane elastically and bounces as shown in figure below:

Find the distance covered by ball along the incline plane from first hit to 4th bounce.
1. From the figure, let the ball break off when the radius vector makes an angle \( \theta \) with the vertical.

From the free body diagram,

\[
mg \cos \theta - N = \frac{mv^2}{(R + r)}
\]

When the ball breaks

\[
N = 0
\]

i.e.

\[
g \cdot \cos \theta = \frac{v^2}{(R + r)} \quad \text{...(i)}
\]

Loss of the potential energy of the ball,

\[
mg \cdot (R + r) - mg \cdot (R + r) \cos \theta
\]

Gain in kinetic energy of the ball,

\[
\frac{1}{2}m v^2 + \frac{1}{2}k \omega^2
\]

\[
= \frac{1}{2}m \omega^2 + \frac{1}{2} \frac{2}{5} m \omega^2
\]

\[
= \frac{7}{10} m \omega^2
\]

Therefore, by laws of conservation of energy, we have

\[
mg \cdot (R + r)(1 - \cos \theta) = \frac{7}{10} m \omega^2
\]

\[
g(1 - \cos \theta) = \frac{7 \omega^2}{10(R + r)}
\]

From Eqs. (i) and (ii), we get

\[
g \cos \theta = \frac{10}{7} \Rightarrow \cos \theta = \frac{10}{17}
\]

From Eq. (i), we get

\[
g \frac{10}{17} \cdot \frac{5^2}{R + r} \Rightarrow \omega^2 = \left[ \frac{10g}{17} \frac{R + r}{r^2} \right]
\]

\[
\Rightarrow \omega = \sqrt[3]{\frac{10g}{17r^2} \frac{R + r}{R + r}}
\]

2. \( LM \) is the wall and \( O \) is the centre of the sphere.

(i) Its weight \( W \) acting vertically downwards at \( O \).

(ii) The normal reaction \( R \) between the sphere and the wall at the point of contact \( M \) acting normally to the wall, passing through the centre \( O \).

(iii) The tension \( T \) in the string \( LN \).

Since, two forces \( R \) and \( W \) are meeting at \( O \), therefore, the third force \( T \) is also passing through \( O \).

Let \( \theta \) be the inclination of the string to the vertical. Also, we are given that (string \( LN \) = radius \( ON \)).

\[
OL = (ON + NL) = 2ON
\]

\[
\therefore \text{In } \triangle OML, \quad \sin \theta = \frac{OM}{OL} = \frac{OM}{2ON} = \frac{1}{2} \quad \text{...(i)}
\]

Applying Lami’s theorem at \( O \), we get

\[
\frac{R}{1} = \frac{W}{\cos \theta}
\]

\[
\Rightarrow \frac{R}{W} = \cos \theta
\]

From Eq. (i), we get

\[
\sin \theta = \frac{1}{2}
\]

\[
\cos \theta = \frac{\sqrt{3}}{2} \quad \text{and} \quad \tan \theta = \frac{1}{\sqrt{3}}
\]

Therefore, from Eq. (ii),

\[
T \frac{W}{\cos \theta} = \frac{2W}{\sqrt{3}} \quad \text{and} \quad R = W \tan \theta = \frac{W}{\sqrt{3}}
\]

3. As, the force \( F \) is applied on the block of mass \( m_2 \) it shifts the block towards right (if \( F \) exceeds the friction force acting on the block). This process elongates the spring. So the restoring force generates in the spring and tends to move the block of mass \( m_1 \). If this restoring force exceeds the limiting frictional force the block of mass \( m_1 \) moves.
For mass $m_1$,
For vertical equilibrium,
\[ m_1 g = N_1 \]
and for horizontal motion to impend,
\[ kx_0 \geq f_1 \]
\[ f_1 = \mu N_1 \]
\[ \Rightarrow kx_0 \geq \mu m_1 g \]
but $x_0$ is minimum elongation,
\[ kx_0 = \mu m_1 g \]
\[ \Rightarrow f_0 \leq \mu N_1 \]
\[ \Rightarrow 1/2 mg \leq \mu m_1 g \text{ and } m_1 \geq (4mg) \]
\[ \Rightarrow M \geq 1/2 \text{ and } M \geq 1/4 \]
Combining, we have
\[ M \geq 1/2 \text{ and } M \geq 1/4 \]
\[ \ldots (i) \]
\[ \text{Also, } mg - 2T = ma \]
\[ T = \mu mg = ma \]
\[ \Rightarrow a = 1 - 2\mu g \text{ and } \]
\[ T = 2 + \mu mg \]
\[ \text{For } A, \]
\[ N = 4mg \]
\[ f = f_1 + T \]
\[ \Rightarrow f = \mu mg + 2 + 6\mu mg \]
\[ \Rightarrow \mu < 1/2 \text{ and } \mu \geq 1/7 \]
Combining all results, we have
\[ \mu \geq 1/7 \text{ and } \mu < 1/2 \]
\[ \Rightarrow 1/2 > \mu \geq 1/7 \]
6. In vertical motion of ball $a_y = g \cos \theta$
Horizontal component of initial velocity of ball is $v_x = g \sin \theta$
\[ t_1 = \text{time in which ball falls through height } h. \]
\[ \Rightarrow t_1 = \frac{2h \cos \theta}{g \cos \theta} = \frac{2h}{g} \]
As the ball makes 3 hits and each hit takes a time $t = 2t_1$, we have time duration between first and 4th hit is
\[ t_4 = \left(2 \frac{2h}{g}\right) \times 3 = 6 \frac{2h}{g} \]
Total distance covered down the plane is
\[ s = ut + \frac{1}{2} a t^2 \]
\[ \Rightarrow s = v_x t_4 + \frac{g \sin \theta}{2} \times t_4^2 \]
\[ \Rightarrow s = (\frac{g \sin \theta}{2} \frac{2h}{6} \frac{2h}{g} + \frac{g \sin \theta}{2} \left(\frac{2h}{g}\right)^2 \]
\[ \Rightarrow s = 12h \sin \theta + 36h \sin \theta = 48h \sin \theta \]
1. What will be the expression for the deviation in angle between a stationary plumb line and the local vertical at a latitude $\phi$ and calculate the maximum value of this deflection? Assume, the earth to be a uniform sphere of radius 6400 km.
   (a) $\frac{\omega R^2 \sin 2\phi}{2g}$
   (b) $\frac{\omega R \sin 2\phi}{2g}$
   (c) $\frac{\omega^2 R \sin 2\phi}{g}$
   (d) $\frac{\omega^2 R^2 \sin 2\phi}{2g}$

2. A coil of wire consisting of twenty turns in the shape of equilateral triangle of side 5 cm is placed in a magnetic field of $10^{-3}$ T parallel to the plane of the triangle. What is the torque acting on the coil when a current of 0.1 A is allowed to flow?
   (a) $4 \times 10^{-3}$ N-m
   (b) $2.2 \times 10^{-6}$ N-m
   (c) $8 \times 10^{-4}$ N-m
   (d) $9 \times 10^{-5}$ N-m

3. One model rocket is a trolley on which several spring launchers are installed. Each spring is compressed and therefore stores $E = 100$ J of elastic energy. The system, whose total mass is $m = 100$ kg is initially at rest. Find the velocity of the trolley, if the structure shoots out three balls with mass $m = 5$ kg in succession and in the same direction along the longitudinal axis.
   (a) 1.05 m/s
   (b) 1.03 m/s
   (c) 0.80 m/s
   (d) 1.09 m/s

4. The circuit shown in the figure is connected to an AC generator that supplies 5 V. $R = 5 \, \text{k}\Omega$ and the two capacitors are identical. The ammeter reads 1 mA while the voltmeter reads 13 V. What will be metres read if the angular frequency of the generator is changed from $\omega$ to $\omega/\sqrt{2}$? Assume the metres to be ideal.
   (a) 0.512 mA and 8.91 V
   (b) 0.630 mA and 4.35 V
   (c) 0.305 mA and 6.10 V
   (d) 0.404 mA and 7.15 V

5. Determine the mass ratio of deuterium and hydrogen, if their $H_2$ lines have wavelengths of 6561.01 Å and 6562 Å (It was thorough measurement of this type that deuterium was discovered)
   (a) 1.0
   (b) 2.0
   (c) 4.0
   (d) 6.0

6. The telephone wire of diameter 1 mm is suspended parallel to the ground at a height of 10 m. What is the capacitance to ground of this wire per unit length? (Assume the ground to be a conducting plane)
   (a) $7 \times 10^{-6} \text{F/m}$
   (b) $5.2 \times 10^{-6} \text{F/m}$
   (c) $8.2 \times 10^{-6} \text{F/m}$
   (d) $8.4 \times 10^{-5} \text{F/m}$

7. Estimate the minimum frequency of a $\gamma$-ray that causes a deuteron to disintegrate into a proton and a neutron, commenting on any assumptions we make. The masses of the particles are $m_d = 2.0141 \, m_p$, $m_p = 1.0078 \, m_n$, $m_n = 1.0087 \, m_p$.
   (a) $5.4 \times 10^{10}$ Hz
   (b) $6.9 \times 10^{11}$ Hz
   (c) $6.2 \times 10^6$ Hz
   (d) $7.3 \times 10^4$ Hz

8. Consider the elastic scattering of a photon of frequency $\nu$ by a stationary electron (the Compton effect). Find the energy of a photon of initial energy 1 MeV after a single $180^\circ$ scattering.
   (a) 0.50 MeV
   (b) 0.20 MeV
   (c) 0.75 MeV
   (d) 1.25 MeV

9. A particle of mass $m$ situated inside a deep one-dimensional potential well of width $a$ described by $V = 0$ for $0 \leq x \leq a$ and $V = \infty$ elsewhere. An electron is confined within a thin layer of a semiconductor. If the layer can be treated as a deep one-dimensional well, calculate its thickness if the difference in energy between the first and second levels is 0.05 eV.
   (a) 4.8 nm
   (b) 9.2 nm
   (c) 10.1 nm
   (d) 15.3 nm
10. An atom has energy level \( E_n = -\frac{A}{n^2} \), where \( n \) is an integer and \( A \) is a constant. Among the spectral lines that the atom can absorb at room temperature, are two adjacent lines with wavelengths 97.5 nm and 102.8 nm. Find the value of the constant \( A \) in electron volt.

- (a) 12.6 eV
- (b) 13.6 eV
- (c) 16.2 eV
- (d) 17.4 eV

---

**Concept**

**Radiation**

**Perception** When two bodies at temperature \( T_1 \) and \( T_2 \) (\( T_2 > T_1 \)) are placed in vacuum at a certain separation, they will never attain equilibrium.

**Explanation** This is very common misconception among students. Their thought in this regard is that as there is no medium present, hence they will never attain equilibrium because of absence of common medium. But here condition is not so. Here, transfer of energy will take place by radiation and rate of loss of heat will be given by Stefan's law. Hence rate of heat gained by the colder body will be proportional to \( T_2^4 - T_1^4 \).

---

**Concept** Work done on a Gas in a Cyclic Path

**Perception** When a cyclic process is traced in anti-clockwise sense, net work is done by the gas.

**Explanation** When a cyclic path is traced in anti-clockwise direction, compression work is more than the expansion work.

(As show, \( W_{AB} = \text{area under AB line} \) is more than \( W_{CD} \))

So, net work is done on the gas and is negative.
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